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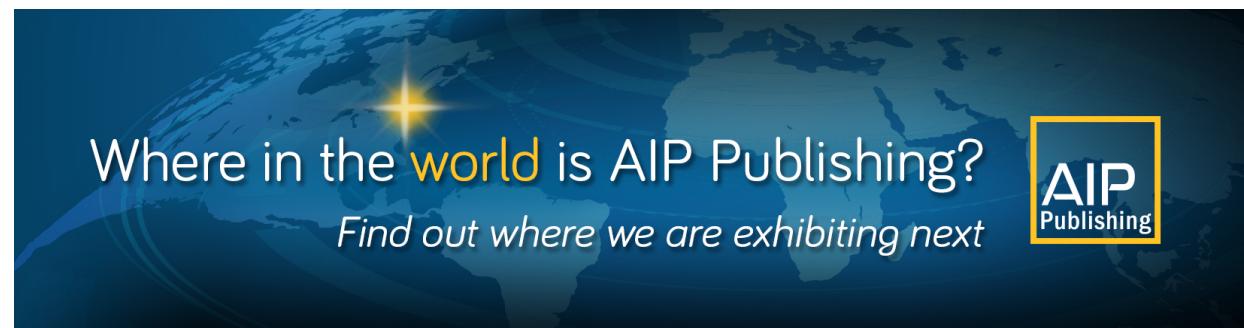
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Elastic Properties of Metals and Alloys. II. Copper

H. M. Ledbetter and E. R. Naimon[†]

Cryogenics Division, Institute for Basic Standards, National Bureau of Standards, Boulder, Colorado 80302

The elastic properties of copper have been compiled and reviewed. Polycrystalline elastic constants included are: Young's modulus, the shear modulus, the bulk modulus, and Poisson's ratio. Single-crystal constants of second-, third-, and fourth-order are included. Over 200 references to the experimental literature are given. A few theoretical numbers are included. When sufficient data exist, best values are recommended together with their standard errors. Effects on the elastic constants of temperature, pressure, and mechanical (plastic) deformation are included. The Cauchy (central-force) relationships and the single-crystal—polycrystal relationship are also discussed.

Key words: Bulk modulus; compressibility; copper; elastic constants; Poisson's ratio; shear modulus; single-crystal elastic coefficients; Young's modulus.

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1. Introduction

The purpose here is to present a compilation and a review of the elastic properties of copper. A similar study was reported by Ledbetter and Reed (1973) for iron, nickel, and iron-nickel alloys. That reference also contains more complete discussions on some aspects of elasticity that are relevant here. These include: crystal physics and thermodynamics of elastic constants; effects of temperature, pressure, etc.; and the relationship between elastic constants and other physical parameters, including the Debye temperature.

Copper is especially interesting to metal physicists and to engineers; because of its electronic structure, copper approximates a standard metal [Pippard (1960)]; it is used widely technologically, elementally and alloyed. Both single crystal and polycrystalline elastic constants are included, and their interrelationship is discussed. These constants relate directly to interatomic forces in solids. The polycrystalline constants are Young's modulus, the shear modulus, the bulk modulus (reciprocal compressibility), Poisson's ratio, etc. These are the constants used to characterize most engineering materials.

More than two-hundred references are given to the experimental literature. The earliest elastic study on copper was reported by Wertheim (1848). The first cryogenic (87 K) study was by Benton (1903). The first high-temperature data were reported by Kikuta (1921), who measured the shear modulus from 298 to 1126 K. The highest temperature measurements were made by Köster (1948a, 1948b) for Young's modulus and by Koch and Dannecker (1915) for the shear modulus, 1230 and 1273 K, respectively.

Copper's second-order elastic constants (SOEC) were reported first by Goens (1933) and Kimura (1933). Subsequently, seventeen additional sets of SOEC have been reported; nine of these included effects of temperature, three included pressure, and nine included alloying. A summary of experimental conditions for single-crystal studies is given in table 1.[1] Experimental second-order elastic stiffnesses are given in table 2.

Third-order single-crystal elastic constants (TOEC) were reported first by Hiki and Granato (1966); subsequently, five additional reports on third-order constants have been made. Experimental third-order elastic stiffnesses are given in table 3.

Thus, the elastic proptrties of copper are probably better characterized than those of any other metal.

2. Electronic Structure

A symbiotic relationship exists between the studies of the elastic properties of metals and the electron theory of metals. Calculations of elastic constants are convenient and valid checks on proposed interatomic potentials. Observed elastic anomalies often provide a point of departure for extending the theoretical description of metals.

The elastic properties of copper are strongly affected by its electronic structure, [Ar] $3d^{10}4s^1$.[2] Copper's ion cores consist of an Ar rare-gas configuration plus a complete d shell. Thus, the outer-core electrons are loosely bound, and the ion-core volume is large relative to the atomic volume. Little difference exists between the valence ($4s$) electrons and the outer-core ($3d$) electrons. Thus, many equilibrium properties should be determined mainly by ion-ion overlap; the Fermi-gas pressure due to valence-electron interactions is secondary. These equilibrium properties include the interatomic spacing and the bulk modulus. Since the core electrons are poorly shielded from the valence electrons, a strong valence-electron—ion-core attractive interaction occurs in copper. Effectively, the $3d$ electrons create an additional potential well. Thus, the electron-ion system tends to contract because the $4s$ electrons tend to occupy the well. Opposing this minimization of free-electron energy are the repulsive ion-ion interactions. Because of their strong radial dependence, these interactions contribute progressively more to higher derivatives of the energy. Whether ion cores are "hard" or "soft" affects energy only incidentally, but perfectly "hard" spheres would determine precisely the interatomic spacing and would have an infinite bulk modulus. This valence-electron—ion-core interaction accounts roughly for the relatively high cohesion and high bulk modulus exhibited by copper.

Simple theories predict cohesive energies for copper that are too low. Brooks (1958) surmised that the extra cohesion is due to correlated motions of d-shell electrons and that these motions arise similar to van der Waals interactions. Overlap of the d shells makes such a model difficult to describe theoretically.

Elastic properties are related intimately to the problem of cohesion since elastic stiffness coefficients are spatial derivatives of the interatomic potential, while cohesive energy is related to the depth of the potential well. Usually, solids with higher cohesive energies also have higher elastic stiffness.

3. Theoretical Calculation of Elastic Constants

In a landmark paper, Fuchs (1936) was first to study the fundamental problem of cohesion in copper. He found that exchange and correlation interactions between $4s$ and $3d$ electrons are important in copper, as are the ion-ion repulsive interactions. Because of the difficulties of treating an eleven-electron problem, Fuchs considered the $3d$ electron band to be rigid. The ion-ion repulsive interaction was treated by a modified Thomas-Fermi approach. Fuchs' calculations of the shear constants of copper were quite successful, obtaining $C_{44} = 0.89$ and $C' = 0.26$, in units of 10^{11} N/m^2. Experimentally, $C_{44} \approx 0.82$ and $C' \approx 0.26$ at 0 K as shown below in the figures for the temperature dependences of the C_{ij}.

Subsequently, numerous theoretical calculations of the elastic constants of copper have been reported. The most

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[1] Tables have been placed at the end of this paper.

[2] This particular electronic-state assignment is not unique. For example, in the Engel-Brewer theory of bonding, copper is considered to have an [Ar]$3d^8 4s^1 p^2$ electronic structure. For present purposes the important point is that the ion core of copper is large and stable.

recent calculations have used pseudopotentials and either the method of homogeneous deformation (after Fuchs) or the method of long waves (after Born). These calculations have been made by Srinivasan and Girirajan (1973) and by Thomas (1973). These calculations reveal many important elastic properties of copper that are verified experimentally. Some of these are discussed below.

4. Second-Order Elastic Constants

The second-order elastic constants of a solid describe its linear elastic behavior and are directly related to the harmonic part of the interatomic potential. Thus, the SOEC play an important role in determining the nature of forces in solids. Besides being of fundamental interest, SOEC are often useful parameters in engineering design.

The earliest measurements of SOEC were performed by static methods, which yield the isothermal elastic constants. Quantities such as Young's modulus, the shear modulus, and Poisson's ratio were measured for single crystals of various orientations. These moduli are related to the single-crystal SOEC by simple mathematical transformations (see section 9).

Most present-day measurements of SOEC use dynamic methods. The advantages of dynamic methods include smaller specimen size, simpler temperature control, avoidance of creep, and higher precision. Dynamic methods yield the adiabatic elastic constants. However, adiabatic and isothermal moduli are related by well-known thermodynamic equations. It is usually found that the difference between adiabatic and isothermal elastic constants is a few percent or less, which is small compared to the error in static (isothermal) measurements. The difference for C_{11}, C_{12}, and $\frac{1}{3}(C_{11}+2C_{12})$ is $9\alpha^2 B_T T/(\rho C_v)$, where α is the linear thermal-expansion coefficient, B_T is the isothermal bulk modulus, T is the temperature, ρ is the mass density, and C_v is the specific heat. For copper at room temperature this correction is 3.0 percent for the bulk modulus. There are no corrections for the shear moduli $\frac{1}{2}(C_{11}-C_{12})$ and C_{44}.

Earlier dynamic measurements were performed in the kilohertz region, where a standing-wave resonance was established in a system composed, at least partly, of the specimen under investigation. Higher precision than that of static methods was obtained from the measurement of a resonance frequency rather than a small displacement. These methods were reviewed by Read, Wert, and Metzger (1959).

The most widely used method today employs frequencies near 10 MHz. A piezoelectric tranducer is cemented to one of two parallel faces of the specimen. An ultrasonic pulse of ~ 1 μs duration is transmitted through the specimen. The transit time of the pulse is related to the ultrasonic velocity, which in turn is related to the adiabatic moduli. Interferometric techniques have made possible very high-precision measurements of ultrasonic velocities. Absolute accuracies of better than 0.1 percent and relative accuracies of 10^{-6} or better are possible. Such precision results in better determinations of temperature and pres-

sure dependences of the SOEC. A review of ultrasonic studies of properties of solids, including various measurement techniques of ultrasonic velocities, was given recently by Fuller, Granato, Holder, and Naimon (1974).

The SOEC of copper have been determined statically and dynamically. Despite limited precision, some of the static-method results (especially those of Goens (1933)) are in excellent agreement with the more modern dynamic results. This is shown in table 2 where the static-method results are included and are labeled isothermal.

5. Cauchy Relationships

Because the Cauchy relationships are not satisfied in most metals, including copper, it follows that the interatomic forces are noncentral. Cauchy relations are simple relations that can be derived when interatomic interactions are purely central, when all atoms are at centers of symmetry, and when the crystal is stress-free. For cubic crystal symmetry, which copper has, the Cauchy relations for SOEC are $C_{12}=C_{44}$; the TOEC Cauchy relations are $C_{112}=C_{166}$ and $C_{123}=C_{144}=C_{456}$. Hiki and Granato (1966) observed that the TOEC of copper obey the Cauchy relations much more closely than do the SOEC. The interpretation that Hiki and Granato gave to their results was that short-range central forces, in this case arising from d-shell overlap, play a progressively greater role as one progresses from calculations of the energy to the lattice constant and second-, third-, and higher-order derivatives of the total energy. If short-range forces play a dominant role in third-and higher-order elastic constants, then the nearest-neighbor atoms should make the most important contributions. Hiki and Granato showed the following relations hold for copper if only nearest-neighbor central-force interactions are considered:

$$C_{12}=C_{44}=\tfrac{1}{2}C_{11},$$
$$C_{112}=C_{166}=\tfrac{1}{2}C_{111}, \text{ and } C_{123}=C_{144}=C_{456}=0.$$

As shown by the data in tables 2 and 3, the third-order nearest-neighbor relations are satisfied to a much greater extent than are the second-order nearest-neighbor relations. If second-neighbor interactions are considered, then only the geometrical factor of $\frac{1}{2}$ is affected in these interrelationships. The shift from $\frac{1}{2}$ is small if the interatomic potential is short ranged.

6. Third-Order Elastic Constants

Third-order elastic constants (TOEC) are important mainly because they describe the anharmonicity of interatomic potentials. Nonlinear stress-strain behavior is a direct consequence of non-vanishing elastic constants of higher than second order; in fact, as described below, deviations from Hooke's law can be used to determine the values of the TOEC. TOEC have been studied more for copper than for any other metal. These data are summarized in table 3.

Three methods have been used to determine the TOEC of copper, and they are discussed in the following paragraphs.

(a) *Variations of Ultrasonic Wave Velocities due to Hydrostatic or Uniaxial Stresses*

Only method (a) yields a complete set of TOEC; therefore it is used most. Since copper is cubic, it has three independent SOEC (C_{11}, C_{12}, and C_{44}) and six independent TOEC (C_{111}, C_{112}, C_{123}, C_{144}, C_{166}, and C_{456}). SOEC can be determined from ultrasonic wave velocities, and TOEC from stress derivatives of the wave velocities.

Pressure-derivative data alone are insufficient for determining a complete set of TOEC since there are only three independent SOEC pressure derivatives. The following combinations of TOEC can be obtained from hydrostatic-pressure experiments:

$$-(C_{111}+2C_{112}) =3B(1+dC_{11}/dP)+C_{11}, \quad (1)$$

$$-(C_{123}+2C_{112}) =3B(-1+dC_{12}/dP)+C_{12}, \quad (2)$$

$$-(C_{144}+2C_{166}) =3B(1+dC_{44}/dP)+C_{44}, \quad (3)$$

where

$$B=\tfrac{1}{3}(C_{11}+2C_{12}) \quad (4)$$

is the bulk modulus and P is the pressure.

The additional data needed for a complete set of TOEC can be obtained by measuring the effects of uniaxial stresses on wave velocities. When hydrostatic-stress and uniaxial-stress data are combined to obtain a complete set of TOEC. the problem is often overdetermined. (In fact, for cubic symmetry, a complete set of TOEC can be determined from six or more independent uniaxial measurements.) This overdetermination permits a least-squares analysis to be made for the "best" TOEC. Hydrostatic-stress data are more reliable than uniaxial-stress data and should be weighted accordingly in the data analysis. Reasons for this include: (1) dislocations are affected by uniaxial stresses and contribute spuriously to the elastic strain, (2) a pure hydrostatic stress is easier to obtain experimentally than a pure uniaxial stress, and (3) large uniaxial stresses cause plastic deformation while large hydrostatic stresses don't, unless by some mechanism dilatational stress is converted to shear stress.

For copper, the effects of hydrostatic stress on ultrasonic wave velocities were first measured by Lazarus (1949) and later by Daniels and Smith (1958). More recently Debesis (1971) and Ho, Poirier, and Ruoff (1971) investigated the effects of temperature on the hydrostatic-stress derivatives of SOEC of copper. Hydrostatic-pressure derivatives of the second-order elastic constants of copper are given in table 4.

Hiki and Granato (1966) were the first to measure a complete set of TOEC for copper. In fact, the results of Hiki and Granato (who measured the TOEC of silver and gold also) were the first complete set for any metal. They used an ultrasonic interferometric method to detect small wave-velocity changes in crystals stressed hydrostatically and uniaxially. Their success encouraged further studies on copper, such as effects of temperature and alloying on the TOEC. Using only uniaxial-stress measurements, Salama and Alers (1967) determined the TOEC of copper at 295.

77, and 4 K. Their specimen was neutron irradiated to prevent dislocation motion. Salama and Alers (1968) also determined the effect of alloying with nickel on the TOEC of copper; and Debesis (1971) measured the pressure derivatives of SOEC of copper-nickel alloys as a function of temperature. Recently, Cain and Thomas (1973) used hydrostatic-stress and uniaxial-stress measurements to obtain complete sets of TOEC for copper alloyed with aluminum.

(b) *Variations of the Finite-Amplitude Distortion of an Initially Sinusoidal Ultrasonic Wave*

Method (b) involves the propagation of large-amplitude longitudinal ultrasonic waves in a single crystal. Because the material is nonlinear, an initially sinusoidal wave is distorted and harmonics of the fundamental frequency are generated. Measurement of the amplitudes of the fundamental and second harmonics as a function of path length determines certain combinations of TOEC, depending on the crystallographic propagation direction. For cubic crystals with longitudinal waves propagating in the [100], [110], or [111] direction:

$$K_{100} =C_{111}, \quad (5)$$

$$K_{110} =\tfrac{1}{4}(C_{111}+3C_{112}+12C_{166}), \quad (6)$$

$$K_{111} =1/9(C_{111}+6C_{112}+12C_{144}+24C_{166} \quad (7)$$
$$+2C_{123}+16C_{456}).$$

The K's are called nonlinearity parameters and have been measured in copper at room temperature by Gauster (1966) and by Gauster and Breazeale (1968), and as a function of temperature by Peters, Breazeale, and Paré (1970). This method avoids applying stresses to the crystal, but requires supplementary data from other experiments to determine a complete set of TOEC. Gauster and Breazeale (1968) reported complete sets of TOEC of copper by combining their results with the pressure-derivative data of other workers.

If method (b), which involves second-harmonic generation, is used to measure TOEC, dislocation effects must be eliminated since dislocations also lead to the generation of second harmonics. To date, method (b) has had few applications. A summary of the nonlinearity parameters K of single-crystal copper is given in table 5.

(c) *Deviation from Hooke's Law in Static Experiments*

Method (c) has been used to measure deviations from Hooke's law of single-crystal whiskers of copper. The nonlinear stress-strain relationship is

$$\epsilon=P/E+\delta(P/E)^2+\zeta(P/E)^3+\ldots, \quad (8)$$

where ϵ is the strain, P is the force per unit undeformed area, and E is Young's modulus for the crystallographic direction of interest. The parameters δ and ζ are nonlinearity constants and represent the nonlinear elastic behavior. A summary of these constants is given in tables 6 and 7. Measurements of δ have been made on copper whiskers with (100), (110) and (111) orientations and the relationships between δ and TOEC have been derived by Seeger

and Buck (1960). It is found that δ_{100} is a function of C_{111} C_{112}, and C_{123}; δ_{100} is a function of all TOEC except C_{456}; and δ_{111} is a function of all six TOEC. Measurements of δ_{hkl} were made by Powell and Skove (1968), by Riley and Skove (1973), and by Kobayashi and Hiki (1973). Because this method does not determine a complete set of TOEC, Powell and Skove (1968) and Riley and Skove (1973) supplemented their data with pressure-derivative data of others in order to report complete sets of TOEC.

Because method (c) produced results that varied from sample to sample, Powell and Skove (1968) suggested that TOEC may be more structure-sensitive than are SOEC. However, Kobayashi and Hiki (1973) found an apparent yield-stress dependence, assumed to arise from defects on the specimen surfaces. Their analysis explained the experimental variations, and the true nonlinearity constants, possessed by crystals without defects, were evaluated.

The TOEC obtained by method (c) differ in principle from those obtained by ultrasonic methods. Method (c) involves static measurements, and the resulting TOEC are isothermal stress derivatives of *isothermal* SOEC. Ultrasonic techniques are dynamic, and the resulting TOEC are "mixed," being isothermal stress derivatives of *adiabatic* SOEC. Expressions relating the purely isothermal TOEC to the experimentally observed quantities have been derived by Powell and Skove (1967). Usually the differences between isothermal and "mixed" constants are within experimental error limits; therefore, the differences can usually be neglected.

The conclusion by Hiki and Granato (1966) that the ion-ion overlap energy should contribute dominantly to the TOEC of copper was recently verified by Thomas (1973), who used a pseudopotential model to calculate the TOEC of copper. Thomas employed the method of homogeneous deformation and obtained good agreement with experiment. A similar calculation was performed by Srinivasan and Girirajan (1973), who used the method of long waves and a pseudopotential model. The results also agree well with experiment and again indicate the dominant contribution of ion-ion overlap to TOEC of copper. Theoretical values for third-order elastic constants are included in table 3.

Further discussion, both theoretical and experimental, on nonlinear elastic properties can be found in Green (1973) together with many valuable references to the literature on this topic.

7. Fourth-Order Elastic Constants

The accuracy involved in static tests is usually much less than that in ultrasonic measurements. Also, the nonlinearity constants δ are relatively sensitive to small variations in SOEC and TOEC; a 1 percent change in SOEC can cause changes in δ as large as 10 percent. However, static tests provide higher strains and thus permit semi-quantitative estimates of fourth-order elastic constants (FOEC) to be made.

Limited results have been obtained for the FOEC of

copper. Hiki, Thomas, and Granato (1967) used a continuum model to derive expressions for the temperature dependence of the three SOEC. These expressions were in terms of second-, third-, and fourth-order elastic constants. Using available experimental data for SOEC, TOEC, and temperature derivatives of SOEC, they deduced information on certain FOEC. Their results agreed fairly well with those calculated earlier by Rose (1966). Further information on FOEC of copper was obtained by Riley and Skove (1973), who measured the nonlinearity constant ζ for copper whiskers with $\langle 100 \rangle$, $\langle 110 \rangle$, and $\langle 111 \rangle$ orientations. This information provides three independent linear equations in the eleven FOEC for cubic symmetry. Using the theoretical FOEC of Rose (1966) and the experimental SOEC and TOEC, Riley and Skove (1973) also calculated ζ and compared with their measured values. Order-of-magnitude agreement was obtained. Fourth-order elastic constant data are given in table 8.

8. Polycrystalline Elastic Constants

For technological uses, metals are almost always in polycrystalline form. Polycrystals (aggregates of single crystals, or crystallites) can usually be described by relationships for isotropic materials; some exceptions are noted in section 9. Several constants are used to describe the elastic behavior of isotropic materials. While only two of these are independent, the others are useful for simply describing a particular load-displacement or stress-strain situation.

For convenience, the most common constants are described briefly here. Young's modulus E is appropriate to the case of uniaxial loading and is given by

$$\sigma = E \epsilon, \tag{9}$$

where σ is the stress (force per unit area), ϵ is the strain (fractional length change), and both are measured along the loading axis. The shear modulus G is appropriate to torsion, twisting, or pure or simple shear and is given by

$$\tau = G \gamma, \tag{10}$$

where τ is the stress (tangential force per unit area) and γ is the angle of shear. The bulk modulus B is appropriate to a uniform (hydrostatic) pressure and is given by

$$P = -B \frac{\Delta v}{v}, \tag{11}$$

where P is the pressure and $\Delta v/v$ is the volume strain (fractional volume change). Poisson's ratio v measures, for the case of uniaxial loading, the ratio of transverse strain to longitudinal strain and is given by

$$v = -\frac{\epsilon_{transverse}}{\epsilon_{longitudinal}}. \tag{12}$$

From physical arguments it can be deduced that E, G, and B are always positive and that v has limits of -1 and $+\frac{1}{2}$. Negative values of v have never been reported for isotropic materials. In some engineering literature the reciprocal $m = 1/v$ is used. For metals v is typically near a value of $\frac{1}{3}$. If $v \approx \frac{1}{3}$, then $B \approx E$ and $G \approx \frac{3}{8}E$. These approximate relationships are also typically observed in metals. As shown by the data in tables 9 and 10, for copper $v = 0.35$,

$B = 1.13 \, E$, and $G = 0.37 \, E$; thus, copper is typically behaved in the relative values of its elastic constants.

9. Relationship between Single-Crystal and Polycrystal Elastic Constants

The linear elastic behavior of both isotropic and quasi-isotropic materials is described completely by two constants. These two elastic constants cannot be chosen uniquely. For example, an isotropic material's elastic properties can be described completely by Young's modulus E and Poisson's ratio ν. They can be characterized equally well by the bulk modulus B and shear modulus G. Of the four material constants E, G, B, and ν, only two are independent. The relationships among E, G, B, and ν are

$$E = 9GB/(G + 3B), \tag{13}$$

and

$$\nu = \tfrac{1}{2}(3B - 2G)/(3B + G). \tag{14}$$

For present purposes B and G will be considered to be the two independent constants. Other material elastic constants, such as Lamè's constants λ and μ, are also simply related to B and G: $\lambda = B - \tfrac{2}{3}G$ and $\mu = G$.

If a polycrystalline aggregate consists of small crystallites oriented randomly, then it is elastically quasi-isotropic and its elastic behavior can be described by the above relationships. (Quasi-isotropic means macroscopically isotropic, but microscopically anisotropic; an aggregate of anisotropic crystallites is a good example of a quasi-isotropic material. The importance of distinguishing between isotropy and quasi-isotropy has been emphasized by Ledbetter (1973), for example.) In practice, aggregates may have preferred orientations of crystallites or crystallite sizes that are large with respect to the overall aggregate size; in this case the above equations do not strictly apply; the degree of their invalidity depends on the elastic anisotropy (defined below) of the crystallites in addition to the factors of crystallite orientations and sizes.

In principle the quasi-isotropic constants are related to the single-crystal elastic constants. For a cubic material such as copper, this means relating B and G to C_{11}, C_{12}, and C_{44}, that is determining B (C_{ij}) and G (C_{ij}). For cubic symmetry B (C_{ij}) is uniquely determined,

$$B = \tfrac{1}{3}(C_{11} + 2C_{12}). \tag{15}$$

This relationship results because the bulk modulus of a cubic material is a scalar invariant of the elastic constant tensor. Equation (15) assumes, of course, that crystallite boundaries *per se* contribute nothing to the aggregate elastic properties. The validity of this assumption is borne out by the equality (within experimental error) of the polycrystalline and single-crystal bulk moduli of copper as shown in table 10.

The problem remains then to relate the quasi-isotropic shear modulus G to the single-crystal elastic moduli C_{ij}. No unique relationship exists. Since G concerns only shear (not dilatational) deformations, it is expected that only those C_{ij} related to shear deformations would enter in

the expression for $G(C_{ij})$. For cubic materials there are two shear constants, C_{44} and $C' = \tfrac{1}{2}(C_{11} - C_{12})$. Thus, G should depend somehow on these two constants. Physical reasoning requires that the value of G lie somewhere between the values of C_{44} and C'. If it should happen that $C_{44} = C'$, then G will have the same value. In this case the material is elastically isotropic and G is determined unambiguously. A measure of the anisotropy, or deviation from isotropy, is given by

$$A = C_{44}/C', \tag{16}$$

where A is known as the Zener anisotropy. Copper has a relatively high elastic anisotropy; from table 2 it follows that $A = 3.2$ at room temperature. The problem of determining $G(C', C_{44})$ can be reduced to finding the product of C' and a function of the anisotropy. This factorization permits general tabular or graphical solutions for G/C' as a function of A.

By assuming homogeneous strain in an aggregate Voigt (1889) was the first to relate G to the C_{ij}. He obtained

$$G_{\mathrm{V}} = \tfrac{1}{5}C'(2 + 3A). \tag{17}$$

The other extreme, namely homogeneous stress in an aggregate, was assumed by Reuss (1929), who obtained

$$G_{\mathrm{R}} = 5C'\frac{A}{3 + 2A}. \tag{18}$$

Obviously $G_{\mathrm{V}} = G_{\mathrm{R}}$ in the isotropic limit of $A = 1$. Later, Hill (1952) showed that the quasi-isotropic shear modulus G should be bounded by G_{V} and G_{R}, that is

$$G_{\mathrm{R}} \leqslant G \leqslant G_{\mathrm{V}}. \tag{19}$$

Hill suggested an averaging of G_{V} and G_{R} to obtain an effective shear modulus. An arithmetic average gives

$$G_{\mathrm{VRH\text{-}a}} = \tfrac{1}{2}(G_{\mathrm{V}} + G_{\mathrm{R}}), \tag{20}$$

and a geometric average gives

$$G_{\mathrm{VRH\text{-}g}} = \sqrt{G_{\mathrm{V}}G_{\mathrm{R}}}. \tag{21}$$

Hill's method has been adopted widely and is referred to as the VRH approximation. Usually the arithmetic mean is used.

A variety of methods have developed for improving the VRH method. These include those by Hershey (1954), Kröner (1958), Eshelby (1961), Hashin and Shtrikman (1962), Aleksandrov (1966), and Kröner (1967). For cubic symmetry all of these methods predict a shear modulus higher than a VRH-a value. For copper, as shown in table 10, the $G_{\mathrm{VRH\text{-}a}}$ value is already higher than the observed shear modulus polycrystalline copper. Thus, the above methods give values for G that are even further in error.

An averaging method developed recently by Ledbetter and Naimon (1974) does result in an effective shear modulus that agrees much more closely with the observed polycrystalline value. This method assumes that the elastic Debye temperature θ is the same for single crystals and polycrystals of the same material. They derived an expression for G in terms of B, θ and the mass density ρ.

The results in table 10 obtained by the Ledbetter-Naimon averaging method show the close agreement for copper between the predicted value of G and the observed value. That the other quasi-isotropic constants of copper derived from B and G, namely E and ν, also agree well with observed values is an indication, at least for copper, that equating Debye temperatures leads to a better relationship between single-crystal and polycrystal elastic constants.

10. Temperature Dependence of the Elastic Constants

The single-crystal elastic constants of copper behave regularly with respect to temperature. The SOEC continuously decrease with increasing temperature, and for high temperatures $(T > \theta/2)$ the decrease is linear. As the temperature approaches 0 K, the SOEC approach their limiting values with zero slope. The temperature variation of the SOEC of copper is shown in figures 1–7.[3]

The temperature coefficients of the SOEC of copper at room temptrature are listed in table 11. These coefficients are a measure of the relative change in the elastic constants due to temperature effects. As is the usual case for metals, the temperature coefficients of the shear constants (C_{44} and C') are largest in magnitude, being approximately twice the magnitude of the coefficients of the longitudinal constants (C_{11} and C_L). The temperature coefficients of the bulk modulus B and C_{12} are smallest in magnitude.

As indicated in figure 8, data for the temperature dependence of TOEC of copper are less abundant than for the SOEC. Thus, it is more difficult to make definitive statements about the behavior of TOEC with respect to temperature. It does appear, however, that the TOEC do behave essentially linearly with respect to temperature. The constant C_{144} exhibits almost no temperature dependence; the constant C_{456} decreases slightly with temperature; and the constants C_{123}, C_{166}, C_{122}, and C_{111} increase (become less negative) with increasing temperature. The nonlinearity parameters K_{100}, K_{110}, and K_{111} also increase linearly with temperature.

The temperature variations of the quasi-isotropic elastic constants of polycrystalline copper are shown in figures 9–12. The data scatter shown there is much larger than in the single-crystal case. Also shown are curves deduced from averaging single-crystal data and curves obtained from a least-squares fit of a semi-theoretical function to the polycrystalline data. For both Young's modulus E and the shear modulus G, the decrease with temperature is steeper than that deduced from single-crystal data. In fact the magnitude of the temperature coefficient of G is approximately twice that of C_{44} and C'. The most obvious interpretation of this effect is that grain boundaries "soften" the elastic moduli at higher temperatures, especially the shear modulus. The bulk modulus B also seems to decrease faster than the single-crystal curve, while Poisson's ratio parallels the single-crystal curve. (ν increases with increasing temperature, which is regular behavior for Poisson's ratio.) The paucity of data for both B and ν as a function of temperature precludes deciding whether these

parameters also have different averaged-single-crystal and polycrystal values. It is probably not significant that all the data of figure 11 (based on relative measurements) lie below the averaged single-crystal data for B. Temperature derivatives of E, G, B, and ν are given in table 12; these were determined by fitting a semi-theoretical function to the polycrystal data after excluding outlying observations, and from a VRH-arithmetic average of the single-crystal data at different temperatures.

11. Alloying Effects

While the principal object of this study was the elastic properties of unalloyed copper, it seems useful to at least indicate what studies have been made to determine the effect of alloying on the elastic properties of copper. This is done in table 1 for single crystals and in table 13 for polycrystals, and is included as a possible aid to those interested in the wide variety of properties that can be achieved in copper alloys. The elastic properties of copper-zinc alloys, including all of the solid-solution phases, is the subject of a compilation and review now in progress by the present authors.

12. Mechanical-Deformation Effects

Mechanical deformation affects elastic properties of metals in two ways. First, for both single crystals and polycrystals, plastic deformation alters the elastic constants per se by introducing lattice imperfections (vacancies, interstitials, dislocations) and by creating residual stresses and strains in the deformed material. A second effect arises in polycrystals—texture, a non-random distribution of grain orientations—that makes the polycrystalline aggregate elastically anisotropic. While texture can often be neglected, particularly if the degree of texture is small or if the Zener elastic anisotropy factor is near unity, it is a real property of most aggregates; a random distribution of crystallite orientations is really an idealization.

Texture arises in plastically deformed aggregates because certain lattice planes and directions tend to align with the principal directions of plastic flow. Depending on the type of stress—uniaxial, biaxial, etc.—various types of texture arise. Copper is typical of many f.c.c. metals; it has a [111]–[100] wire texture and a (110)[112] principal rolling texture. While textures are usually studied by x-ray diffraction methods, elastic constant measurements can also provide information on texture since they are sensitive to anisotropy.

The relationship between texture and elastic properties was discussed recently and very comprehensively by Green (1973). Copper was included in his discussion and his references; thus, further discussion on this topic needs to be only cursory.

References to texture studies together with some of the available data are given in table 9. Recrystallization-texture data are included along with deformation-texture data. While few authors have attempted to do so, it should

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[a] Figures have been placed at the end of this paper.

be realized, as emphasized by Bradfield and Pursey (1953), that the quasi-isotropic elastic constants can be determined from measurements on a textured aggregate. Examination of table 9 shows also that while the effect of texture on Young's modulus is well defined, the effect of texture on the shear modulus or on Poisson's ratio has not been studied experimentally. One would expect no effect of texture on the bulk modulus of a cubic metal. .

The effects of mechanical deformation on the elastic constants of polycrystalline copper, untextured, are not well defined. There are several possible reasons for this: the complexity of the plastic deformation process; the ambiguity in defining plastic strain; the distinction, in terms of dislocation models and in terms of observed behavior, between micro, low, medium, and high plastic strains; the different methods for imparting plastic deformation—shear, extension, compression, bending, rolling, etc.; the possible anisotropic effect of the deformation process, for example, uniaxial extension may change the shear modulus differently along the loading direction than along the transverse directions; and last, specimens that are believed to be relatively free of texture may be textured.

Some single-crystal mechanical-deformation-effect data are available; these are given in table 2. The general result shown there is that all of the C_{ij} are either unaffected or reduced slightly by plastic deformation. However, additional detailed studies on single crystals using different deformation modes and considering dislocation models of the deformation process would be useful toward understanding the effects of plastic deformation on elastic properties.

13. Concluding Remarks

Copper is well characterized elastically, perhaps better than any other metal. At room temperature, polycrystalline copper is especially well defined with respect to Young's modulus (standard error = 0.6 percent), the shear modulus (s.e. = 2.6 percent), the bulk modulus (s.e. = 2.8 percent), and Poisson's ratio (s.e. = 2.6 percent). (The standard error of the arithmetic mean is defined as $\sigma/\sqrt{n}$, where σ is the standard deviation and n is the number of observations.) Single-crystal parameters are even better characterized: the second-order elastic stiffnesses C_{11}, C_{12}, and C_{44} all have standard errors of 0.2 percent or less. Despite the difficulty of measurements, the larger third-order elastic stiffnesses of copper (C_{111}, C_{112}, and C_{166}) have standard errors of 4 percent or less. Fourth-order elastic stiffnesses of copper are known approximately.

Not only has copper been much studied elastically, but its elastic properties exhibit an unusually ideal behavior. Anomalies in elastic constants due to magnetic transitions, crystal-structure transitions, etc. are absent in copper. The temperature behavior of copper's elastic moduli is classic—continuously decreasing with increasing temperature, relative flatness at low temperatures, zero slope at $T = 0$ K, and linear behavior at high temperatures.

The temperature dependences of the elastic constants of copper are also well characterized. At room temperature,

the temperature derivatives $\dfrac{1}{C_{ij}}\dfrac{dC_{ij}}{dT}$ have standard errors of 6 percent or less. From these, temperature derivatives of E, B, G, and ν can be derived that have only slightly larger errors. This is a more reliable approach than averaging the polycrystalline temperature derivatives to obtain average values of $\dfrac{1}{E}\dfrac{dE}{dT}$, etc.

Quasi-isotropic elastic constants (E, G, B, ν, etc.) determined by averaging single-crystal elastic stiffnesses (the C_{ij}) have lower standard errors than those obtained by measurements on polycrystalline specimens. Thus, the single-crystal data-averaging approach is appealing when highly accurate quasi-isotropic data are desired. However, the attractiveness of this approach is reduced by the multiplicity of suggested averaging methods; the deviations among results obtained by various methods far exceed the combined errors due to imprecisions in the C_{ij} and their compounding due to the averaging calculations.

Recommendations for further work on the elastic properties of copper include: (1) better delineation of the bulk modulus and Poisson's ratio as a function of temperature; (2) further experimental and theoretical studies of higher-order, especially fourth-order, elastic constants since these directly reveal the anharmonic part of the interatomic potential; (3) experimental and theoretical studies of the problem of averaging single-crystal properties to obtain quasi-isotropic properties; (4) careful studies of the effects of mechanical deformation on elastic properties of single crystals; and (5) studies to determine the effect of texture on polycrystalline elastic constants other than Young's modulus.

Acknowledgments

Notes on Tables

Some comments are required for a satisfactory reading of the tables. The following principal notations are used in the tables:

     B = bulk modulus

     C_{ij} = single-crystal elastic stiffnesses (number of subscripts denotes order)

     (d) = deformed plastically

     E = Young's modulus

     f = frequency

     G = shear modulus

     h = hour(s)

     K = degrees Kelvin

   OFHC   oxygen-free high conductivity

P = pressure

rv = relative value

T = temperature

v_l = longitudinal wave velocity $(\rho v_l^2 = B + 4/3G)$

v = Poisson's ratio

ρ = mass density

$*$ = see comment column

Unless indicated by (d) = deformed plastically or by another comment, data can be assumed to represent annealed specimens. Recommended values of elastic constants, given at the bottom of the tables, were obtained by averaging arithmetically, without weighting factors, the data for annealed specimens. All data were included in the average except those deviating by more than $\pm 1.5\ \sigma$ from the mean; σ is the standard deviation, and the standard error of the arithmetic mean $\sigma/\sqrt{n}$ is reported as the imprecision, where n is the number of observations. For a large number of observations (greater than twenty), the 95 percent confidence interval for the arithmetic mean $\bar{x}$ is given approximately by $\bar{x} \pm 2\sigma/\sqrt{n}$. The limits $\bar{x} \pm \sigma/\sqrt{n}$ define a confidence interval of about 70 percent for the arithmetic mean. Other interpretations of the standard error of the mean can be found in standard

works on statistics. Despite large deviations of some observations from the average value in the polycrystalline cases, no systematic trend could be detected. Variants that were considered in attempting to detect a trend include: static versus dynamic measurements and impurity concentrations. Other variables that might be important, but for which convincing data are lacking, include: grain size, grain orientation distribution (texture), segregation of impurities at grain boundaries, and stress-strain states.

Some readers will be interested in compressibility data; compressibility χ is simply the reciprocal of the bulk modulus. The recommended compressibility of copper is $\chi = B^{-1}_{\text{recomm.}} = 0.741 \times 10^{-11}\ \text{m}^2/\text{N}$.

Entries in parentheses were derived from the given data, usually by the formulas

$$v = \frac{E}{2G} - 1,$$

and

$$B = \frac{EG}{3(3G - E)}.$$

These formulas tend to give large errors in v and B, but the numbers are still useful.

Table 1.   Summary of experimental conditions for single-crystal studies;
lattice parameters and densities correspond to room temperature

Reference	Copper Content (%)	Lattice Parameter (Å)	Density (g/cm³)	Method for C_{ij}	Variables Studied				Method for C_{ijk}
					T(K)	P(10^8N/m²)	Alloying	Other	
Goens (1933)				Composite oscillator					
Kimura (1933)				E, G as f (direction)					
Goens, Weerts (1936)					80				
Lazarus (1949)				Ultrasonic (pulse-echo)		0-10			
Jacobsen (1955)	99.99			X-ray scattering					
Long (1955)									
Overton, Gaffney (1955)	Spectroscopic			Ultrasonic (pulse-echo)	4-300				
Daniels, Smith (1958)				Ultrasonic (pulse-echo)		0-10			
Rayne (1958)				Ultrasonic (pulse-echo)			Zn		
Rayne (1959)				Ultrasonic (pulse-echo)	4-300		Zn		
Schmunk, Smith	99.9+	3.6150	8.932	Ultrasonic (pulse-echo)			Ni		
(1961)	99.98 (OFHC)			Ultrasonic (pulse-echo)	4-300		Mn		
Alers, Thompson	99.98 (OFHC), 99.999+			Ultrasonic (pulse-echo)	4-270			Plastic strain, irrad., anneal.	
Epstein, Carlson (1965)	99.9		8.910	Ultrasonic (pulse-echo), resonance			Ni		
Chang, Himmel (1966)	99.999		8.937	Ultrasonic (pulse-echo)	300-800				

Table 1. (cont'd) Summary of experimental conditions for single-crystal studies;
lattice parameters and densities correspond to room temperature

Reference	Copper Content (%)	Lattice Parameter (Å)	Density (g/cm³)	Method for C_{ij}	Variables Studied				Method for C_{ijk}
					T(K)	P(10^8N/m²)	Alloying	Other	
Gauster (1966)									
Hiki, Granato (1966)	99.9995			Ultrasonic (interferometric)					Hydrostatic, uniaxial stresses
Salama, Alers (1967)				Ultrasonic (sing-around)					Uniaxial stress
Gauster, Breazeale (1968)	99.999								Second-harmonic generation
Powell, Skove (1968)									Deviations from Hooke's law
Salama, Alers (1968)									
Riley (1969)									Uniaxial stress
Peters, Breazeale, Paré (1970)	99.999				77-300				Second-harmonic generation
Cain and Thomas (1971)	99.999	3.6147	8.9384	Ultrasonic (pulse-superposition)	275-300		Al		
O'Hara, Marshall (1971)	Extrapolated from alloy data	3.614	8.94	Ultrasonic (pulse-echo)	4-300		Au		
Debesis (1971)	Schmunk, Smith (1960) crystals			Ultrasonic (pulse-echo) buffer rod	77-297	0-4	Ni		Hydrostatic stress
Moment (1972)	Extrapolated from alloy data			Ultrasonic (pulse-echo)			Al, Sn		
Kobayashi, Hiki (1973)									

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Table 2. Second-order adiabatic elastic stiffnesses C_{ij} of copper at room temperature; entries have units of 10^{11} N/m^2

Reference	C_{11}	C_{12}	C_{44}	C'	B	C_L	Comments
Goens (1933)	1.698	1.226	0.753	0.236	1.383	2.215	Isothermal
Kimura (1933)	1.733	1.182	0.612	0.276	1.366	2.069	Isothermal
Goens, Weerts (1936)	1.764	1.292	0.753	0.263	1.449	2.254	Isothermal
Lazarus (1949)	1.710	1.239	0.756	0.236	1.396	2.230	
Jacobsen (1955)	1.70	1.24	0.645	0.23	1.39	2.12	
Long (1955)	1.683	1.221	0.754	0.231	1.375	2.206	
Overton, Gaffney (1955)	1.6839	1.2142	0.7539	0.2348	1.3708	2.2030	
Rayne (1958), (1959)	1.684	1.214	0.755	0.235	1.371	2.204	
Schmunk, Smith (1960)	1.679	1.245	0.7511	0.2332	1.370	2.197	
Waldorf (1960)	1.6897	1.2203	0.7540	0.2347	1.3768	2.2090	
Alers, Thompson (1961)	1.693	1.222	0.753	0.236	1.379	2.210	99.98
(Extrapolated from 270 K)	1.687	1.217	0.750	0.235	1.374	2.202	99.999, annealed
	1.690	1.218	0.753	0.236	1.375	2.207	99.999, annealed, irradiated
	1.666	1.204	0.753	0.231	1.358	2.170	99.999, deformed 5%
	1.680	1.212	0.745	0.234	1.368	2.191	99.999, deformed 5%, irradiated
Epstein, Carlson (1965)	1.697		0.753				[100] orientation, ultrasonic
	1.684	1.212	0.757	0.236	1.369	2.205	[110] orientation, ultrasonic
	1.683						[100] orientation, resonance
						2.213	[110] orientation, resonance
Chang, Himmel (1966)	1.700	1.225	0.758	0.2355	1.385	2.223	
Hiki, Granato (1966)	1.661	1.199	0.756	0.231	1.353	2.186	
Tatm, Thomas (1971)	1.692	1.219	0.754	0.237	1.377	2.209	
O Hara, Marshall (1971)							Extrapolated from alloy data.
Debesis (1971)	1.6960	1.2276	0.7556	0.2342	1.3838	2.2174	
Moment (1972)	1.695	1.223	0.757	0.237	1.380	2.215	Extrapolated from alloy data.
Average values	1.691	1.222	0.7542	0.2346	1.376	2.211	
Standard errors	0.002	0.003	0.0005	0.0005	0.002	0.003	

Table 3. Third-order single-crystal elastic constants C_{ijk} of copper at room temperature; entries have units of 10^{11} N/m^2.

Reference	C_{111}	C_{112}	C_{123}	C_{144}	C_{166}	C_{456}	Comments
Hiki, Granato (1966)	-12.71	-3.14	-0.50	-0.03	-7.80	-0.95	
Salama, Alers (1967)	-15.0	-3.5	-2.5	-1.35	-6.45	-0.16	
Salama, Alers (1968)	-13.90	-7.78	-1.81	-1.40	-6.48	-0.16	
Gauster, Breazeale (1968)	-14.27	-5.21	-1.09	+8.54	-8.36	-5.47	Data supplemented with pressure derivatives of Lazarus (1949).
Gauster, Breazeale (1968)	-14.27	-3.66	-0.98	+0.64	-7.49	+0.44	Data supplemented with pressure derivatives of Daniels, Smith (1958).
Gauster, Breazeale (1968)	-14.27	-7.78	-2.65	-0.06	-7.71	+1.17	Data supplemented with pressure derivatives of Hiki, Granato (1966).
Gauster, Breazeale (1968)	-14.27	-3.87	-1.77	-0.63	-7.44	+0.66	Data supplemented with pressure derivatives of Salama, Alers (1967).
Powell, Skove (1968)	-14.31	-3.47	-1.24	+0.75	-7.50	+0.43	Isothermal; data supplemented with pressure derivatives of Lazarus (1949).
Powell, Skove (1968)	-12.28	-6.34	+0.95	+3.20	-5.74	-1.36	Isothermal; data supplemented with pressure derivatives of Daniels, Smith (1958).
Powell, Skove (1968)	-12.99	-8.11	-1.69	+0.87	-7.96	+0.70	Isothermal; data supplemented with pressure derivatives of Hiki, Granato (1966).
Riley, Skove (1973)	-11.7	-6.5	+1.54	+1.4	-4.8	-1.3	Isothermal; data supplemented with pressure derivatives of Lazarus (1949).
Riley, Skove (1973)	-13.7	-8.6	-0.65	-0.95	-6.6	+0.4	Isothermal; data supplemented with pressure derivatives of Daniels, Smith (1958).
Riley, Skove (1973)	-12.4	-8.2	-1.1	-1.0	-7.0	+0.70	Isothermal; data supplemented with pressure derivatives of Hiki, Granato (1966).
Riley, Skove (1973)	-12.7	-8.2	-0.72	-0.69	-6.7	+0.4	Isothermal; data supplemented with pressure derivatives of Salama, Alers (1968).
Average values	-13.5	-8.0	-1.2	0.66	-7.2	-0.32	
Standard errors	0.3	0.3	0.3	0.7	0.3	0.4	
Theoretical values							
Rose (1966)	-10.40	-7.70	+0.92	+0.92	-7.70	+0.92	Central-force model.
Srinivasan, Girirajan (1973)	-16.08	-8.40	+0.80	-0.02	-8.26	+0.12	Pseudopotential; method of long waves.
Thomas (1973)	-17.02	-9.65	-0.10	+0.34	-8.32	+0.12	Pseudopotential; method of homogeneous deformation.

Table 4. Hydrostatic-pressure derivatives of the elastic constants of copper at room temperature.

Reference	dC_{11}/dP	dC_{12}/dP	dC_{44}/dP	dC'/dP	dB/dP
Lazarus (1949)	4.67	3.53	0.83	0.57	3.91
Daniels, Smith (1958)	6.36	5.20	2.35	0.58	5.59
Hiki, Granato (1966)	5.94	5.19	2.63	0.375	5.44
Salama, Alers (1967)	6.4	5.5	2.5	0.45	5.8
Salama, Alers (1968)	5.92	5.02	2.36	0.45	5.32
Bridgman (1949)					5.3
Rice et al. (1958)					4.1
Al'tshuler et al. (1958)					4.8
Debesis (1971)	6.19	5.08	2.33	0.556	5.45
Ho et al. (1971)	5.95	4.94	2.24	0.50	5.28
Average values	5.92	4.92	2.18	0.50	5.10
Standard errors	0.22	0.24	0.23	0.03	0.20

Table 5. Nonlinearity parameters of single-crystal copper at room temperature; entries have in units of 10^{11} N/m^2.

Reference	$K_{\langle 100 \rangle}$	$K_{\langle 110 \rangle}$	$K_{\langle 111 \rangle}$	Comments
Gauster (1966)	-9.63	-19.48	-16.55	
Gauster, Breazeale (1968)	-14.3	-32.5	-25.9	
Gauster, Breazeale (1968)	-12.7	-32.7	-29.5	Calculated from data of Hiki, and Granato (1966).
Gauster, Breazeale (1968)	-15.0	-29.5	-27.2	Calculated from data of Salama, Alers (1967)
Peters et al. (1970)	-13.9	-31.2	-25.7	
Average values	-13.1	-29.1	-25.0	
Standard errors	0.9	2.5	2.2	

Table 6. Nonlinearity constants δ of single-crystal copper whiskers at room temperature.

Reference	$\delta_{\langle 100 \rangle}$	$\delta_{\langle 110 \rangle}$	$\delta_{\langle 111 \rangle}$	Comments
Powell, Skove (1968)	-3.3	4.8	2.1	
Powell, Skove (1968)	-4.7	10.8	3.8	Calculated from third-order data of Hiki, Granato (1966).
Riley, Skove (1973)	-4.4	8.9	2.1	
Riley, Skove (1973)	-4.6	10.7	3.8	Calculated from third-order data of Hiki, Granato (1966).
Riley, Skove (1973)	-2.2	7.2	2.7	Calculated from third-order data of Salama, Alers (1968).
Kobayashi, Hiki (1973)	-4.3	10.0	3.5	
Kobayashi, Hiki (1973)	-4.51	10.57	3.76	Calculated from third-order data of Hiki, Granato (1966).
Kobayashi, Hiki (1973)	-2.00	7.23	2.68	Calculated from third-order data of Salama, Alers (1967).
Average values	-3.8	8.8	3.1	
Standard errors	0.4	0.8	0.3	

Table 7. Nonlinearity constants ζ of single-crystal copper whiskers at room temperature

Reference	$\zeta_{\langle 100 \rangle}$	$\zeta_{\langle 110 \rangle}$	$\zeta_{\langle 111 \rangle}$	Comments
Riley, Skove (1973)	66 ± 25	184 ± 25	20 ± 10	Static test; deviations from Hooke's law.
Riley, Skove (1973)	70	. . .	. . .	Calculated from SOEC and TOEC of Hiki, Granato (1966) and from theoretical FOEC of Rose (1966).
Riley, Skove (1973)	35	. . .	. . .	Calculated from SOEC and TOEC of Salama, Alers (1968) and from theoretical FOEC of Rose (1966).

Table 8. Fourth-order elastic stiffnesses of copper; entries have units of 10^{11} N/m^2.

Reference	C_{1111}	C_{1112}	C_{1122}	C_{1123}	Comments
Hiki, Thomas, Granato (1967)	101	50.5	50.5	0	Determined from temperature dependence of SOEC assuming nearest-neighbor central forces.
Rose (1966)	74.49	42.33	47.56	-2.62	Central-force model, many-neighbors.

Table 9. Room-temperature elastic properties of polycrystalline copper

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
	10^{11} N/m^2 (except ν)							
Alers, Choi, Liu (1967)	1.09-1.34(d)				Neutron irradiation, rolling direction, rolling temperature	Resonant bar, longitudinal	Cathode copper	Cold worked to 95% reduction
Alers, Liu (1967)	1.16-1.39(d)				Rolling temperature, rolling direction, alloying: Zn	Resonant bar	99.99	Cold worked to 95% reduction
Amagat (1891)	1.21	(0.456)	1.16	(0.350)				B_{Cu}/B_{Fe} = 0.79
Aoyama, Fukuroi (1941)	1.27				T = 78, 289 K		0.95 Ni, 0.0.02 Mn, 0.02 Fe, bal. Cu	
Bennett, Davies (1949)	rv				T = 298-650 K	Resonant bar (0.5-2.0 kHz), transverse	99.98	
Benton (1901)				0.341		Interference apparatus		
Benton (1903)	rv	rv			T = 87 K	Stress-strain		
Bez-Bardili (1935)	1.16	0.429	(1.31)	0.348		Angle of total reflection of ultrasonic waves (1-20 MHz)		ρ = 8.70 g/cm^3 ρv_ℓ^2 = 1.84 x 10^{11} N/m^2
Biller, Helow (1969)	1.27				T = 93-473 K	Resonant bar (9-17 kHz)		
Birch (1937)	1.277	0.468	1.36	0.364	P = 1-4000 atm (~ 0.4G Pa)	Resonant bar	Commercial electrolytic	ρ = 8.904 g/cm^3
Birch, Mallet, de Piaquet (1957)	1.17(d) 1.27				T = 77-873 K, annealing temp.	Resonant bar (8-40 kHz)	99.999	Temp. data given as resonance frequencies
Bock (1954)	*	*		0.346		Simultaneous bending and torsion		*G/E = 0.371
Bordoni (1953), (1954)	1.358(d) 1.373 1.275(d) 1.310				T = 4.5-300 K	Resonant bar (10-50 kHz)	Commercially pure	

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
		10^{11} N/m^2 (except ν)						
Bradfield, Pursey (1953)	1.301	0.480	(1.498)	0.350	Annealing tempera-ture	Longitudinal, torsional vibration (20-150 kHz)	NPL pure	Annealed 3 h at 550°C
	1.296	0.483	(1.364)	0.342				Annealed thoroughly
Bridgman (1923)			1.340(d)		T = 303, 348 K	Pressure cylinder, linear compressibility relative to iron	Commercial rod	
			1.345					
			1.364				99.987	
Bridgman (1949)			1.379			Pressure cylinder, lever piezometer	99.999	
Brouwer (1965)	rv	rv			Plastic strain at T = 70 K	Resonant bar, bending		
Brouwer, Groenenboom-Eygelaar (1967)	rv	rv			Plastic strain at T = 77 K	Resonant bar, bending and torsion		
Bruner, Mecs (1963)	rv				Annealing tempera-ture after plastic deformation at 4 K	Resonant bar		
Bunge, Ebert, Günther (1969)	1.210				Rolling direction, amount of plastic deformation	Resonant bar	99.92	Random specimen
Cabarat, Guillet, LeRoux (1949)	1.248				Alloying: Sn, Zn	Resonant bar		
Chang, Hultgren (1965)			1.348					
Chelnokov (1964)	*				Plastic strain, annealing time	Resonant bar		*Relative resonance frequencies
Collet (1967)		rv			Plastic strain, vibration amplitude, T = 77 K	Torsion pendulum	99.999	
Davis (1943)	1.06(d)				T = 303-508 K	Stress-strain	OFHC	Plastically strained 8%
DeKock, Crans, Druyvesteyn (1965)	rv	rv			Plastic elongation, torsion	Simultaneous bending and torsion		

Table 9. Room-temperature elastic properties of polycrystalline copper (cont'd)

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
	10^{11} N/m^2 (except ν)				
Den Buurman (1970)		rv			Strain amplitude, isochronal anneal	Resonant bar ($\sim$ 280 kHz)	99.999	
Den Buurman, Snoep (1972a)		rv			Strain amplitude, strain cycles (fatigue)	Resonant bar ($\sim$ 170 Hz)	99.999	
Den Buurman, Snoep (1972b)		rv			Strain amplitude, strain cycles (fatigue)	Torsion pendulum	99.999	
Druyvesteyn, Meyering (1941)	1.276	0.473	1.41	0.35	Alloying: Zn	Resonant bar		
Druyvesteyn, Schannen, Swaving (1959)		rv			Plastic torsional strain, annealing	Torsion pendulum	99.999	
Druyvesteyn, Blaisse (1962)	rv				Plastic strain, strain temp., annealing	Lems (1962)	99.999	
Ebert (1935)			1.37		P = 0-5000 kg/cm^2	$\Delta \ell/\ell$ comparative		
Eisner, Ottlyk (1963)	1.32				Applied d.c. potential	Stress-strain	99.99	
Faninger (1969)	1.27	0.480	(1.19)	0.327	Alloying: Ni	Mechanical, x-ray	Commercial electrolytic	
Fantozzi, Boulanger, Gobin (1968)	rv				Plastic strain, annealing	Resonant bar ($\sim$ 2 kHz)	99.999, OFHC	
Fantozzi, Calvet, Gobin (1967)	rv				Plastic strain, isothermal and isochronal annealing	Resonant bar ($\sim$ 2 kHz)	99.93	Grain size $\sim$ 1 mm
Fine (1952)	1.30				T = 233-353 K	Resonant bar		
Förster, Köster (1938), (1937)	1.257					Resonant bar		Annealed 1/2 h at 400°C
Frederick (1947)	1.27	0.47	(1.44)	0.341	T = 78-644 K	Pulse-echo (0.5-15 MHz)		

Table 9. Room-temperature elastic properties of polycrystalline copper (cont'd)

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
		10^{11} N/m^2 (except ν)						
Fukuroi, Shibuya (1950)	1.10				Alloying: Ni	Bending interferometer	Extrapolated from 99 Cu alloy	Annealed 6 h at 800°C
Greer, Bucknall (1964)	1.300				Alloying: Ni, Zn; T = 77-542 K	Resonant bar (∼ 1 kHz)		
Grüneisen (1907a), (1907b)	1.229					Free transverse oscillations, inter-ferometer	Pure	ρ = 8.96 g/cm^3
Grüneisen (1908a), (1908b)	1.226	0.455	1.34	0.352			0.2%As, 0.04%Fe, 1% Ni	
	1.286	0.462	(1.98)	0.391				
	1.279	0.457	1.31	0.368			0.15%As, 0.03%Fe, trace Ni	
Grüneisen (1910)	1.22	(0.48)	1.37	0.35	Bulk modulus: T = 82-438 K			Rod, ρ = 8.96 g/cm^3
	1.25	(0.47)	1.30	0.34				Tube, ρ = 8.89 g/cm^3
Guillet (1939), (1940)	1.24				Alloying: Al, Sn, Zn	Pendulum of LeRolland, Sorin	99.99	
Guye, Schapper (1910)		0.335			T = 78-373 K	Torsional pendulum		
Haskins (1961)					T = 4-350 K	Ultrasonic(2.5 MHz)		Long. vel. = 0.479 x10^6 cm sec^{-1}
Hopkin, Pursey, Markham (1970)	1.30	0.457	1.39	0.340	Alloying: Al, As, Au, Ga, Ge, In, Mn, Ni, Si, Sn, Zn	Resonant bar	99.99	Single-crystal data also
Ide (1936)	1.25	0.459	1.67	(0.362)		Resonant bar, longitudinal		
Iokibe, Sakai (1921)		0.52 / 0.47(d)			T = 298-790 K	Torsional pendulum	Electrolytic purity	Annealed 400°C Hard drawn wire
Jaquerod, Mügeli (1931)	1.260				T = 273-373 K	Torsional pendulum	Commercial	
Joshi, Bhatnagar (1969)		0.450			T = 308-676 K	Torsional pendulum (∼ 70 Hz)	Commercial	
		0.426			T = 301-664 K		Spectroscopic	

Table 9. Room-temperature elastic properties of polycrystalline copper (cont'd)

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
	10^{11} N/m^2 (except ν)							
Joshi, Bhatnagar (1970)	1.207				T – 309-674 K	Flexural resonance (~ 80 Hz)	Commercial purity	
	1.003				T = 303-661 K		Spectroscopic purity	
Kallend, Davies (1971)	1.08-1.36(d)				Rolling direction	Resonant bar	Commercial purity	Initial grain size 0.035 mm, cold rolled to 95% reduction
Kaniak, Müllner (1972)	*				Radiation dosage (α-particles), annealing 78-140 K	Transverse vibration		7-15 μm foils; * gives frequency data, proportional to $E^{1/2}$
Katzenelsohn (1887)		0.359				Torsional pendulum	99.88	
Kawai (1930)	1.20				Plastic strain, annealing temperature	Stress-strain slope	Commercial purity	
	1.24						Electrolytic purity	
Kawai (1931)		0.461	(1.33)	(0.345)	Plastic strain, annealing temperature	Torsion, optical lever	Commercial purity	
Keefer, Sosin (1963)	rv				Isochronal annealing after electron irradiation at 20 K	Resonant bar		
Kiewiet (1886)	1.21	0.431					Electrolytic	
Kikuta (1921)		0.422 0.382(d)			T = 298-1126 K T = 298-1177 K	Torsional pendulum		Annealed wire Drawn wire
Kikuchi (1936)		0.369				Resonant bar		
Kimura (1933)	1.222	0.452	1.366	0.350				
Kisch, Dannecker (1915)		0.416			T – 293-1273 K	Torsional pendulum		
Kohlrausch (1895)	1.17					Torsional pendulum		Average value
Kohlrausch, Loomis (1870)		0.382						

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
		10^{11} N/m^2 (except ν)						
Koiwa, Hasiguti (1967)		*			Isochronal annealing to 433 K after 10% plastic shear strain at 78 K	Torsional pendulum	Zone-refined	*Reports oscillation periods
Korntheuer, Svoboda (1969)	*				Whisker cross-section 5-120 µm^2	Stress-strain		*Varies with whisker diameter, see their fig. 4
Köster (1950)	1.23	0.455	1.36	0.35	Temp. coefficients T = 19.3-1173 K			
Köster (1940)	1.22							
Köster (1948a), (1948b)	1.29				T = 93-1230 K	Resonant bar		
Köster (1953)	1.26			0.34		Longitudinal and transverse deformation	Electrolytic, oxygen-free	
Köster, Rauscher (1948)	1.29				Alloying: Au, Ni, Pd, Pt			
Kralik (1969)	rv				Isochronal annealing after fatiguing at 90 K	Resonant bar (~ 20 kHz)	Electrolytic	
Kuntze (1928)	1.21					Stress-strain		
Kupffer, Zöppritz	1.258							
Kurnakow, Rapke (1914)	1.09					Longitudinal deformation		Annealed wire
Lawson (1941)	rv				Stress	Composite oscillator	Oxygen-free	V_ℓ = 0.367 x 10^6 cm/sec
Leidheiser, Sloope (1970)	1.24				Film thickness 700-5000 Å	Bulge apparatus		E independent of thickness
Lems (1962)	rv				Annealing temperature 83-333 K after plastic strain of 1-9% at 83 K	Resonant frequency (0.9-2.5 kHz)	99.999	1 mm sheet

Table 9. Room-temperature elastic properties of polycrystalline copper (cont'd)

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
	10^{11} N/m^2 (except ν)							
Lems (1964)	rv				Annealing temperature 78-350 K after plastic strain at 78 K	Resonant frequency	99.999	
Lenkkeri, Lähteenkorva (1973)	1.27	0.475	1.37	(0.337)	Alloying: Al	Ultrasonic pulse (10 MHz)	99.99	Grain size = 0.02 mm, ρ = 8.939 g/cm^3
Liu, Alers (1966)	1.04-1.37(d)				Alloying: Zn, rolling direction	Resonant bar	99.99+	ρ = 8.94 g/cm^3
Lobdell, Shinopulos, Fillio (1963)	1.24	(0.469)	(1.17)	0.323		Stress-strain	OFHC	
Lozinskii (1961)	1.237				T = 293-993 K			
Lücke, Roth, Sokolowski (1973)	rv				Strain amplitude, annealing temperature, irradiation temperature (1.5-MeV electrons)	Torsional vibrations (~ 5 kHz)	99.999	< 0.1 mm sheet
Lytton, Kamber, Ardell, Barrett, Sherby (1962)	1.26				Rolling direction; random T=298-563 K; oriented T=298-823 K	Transverse vibrations (250-425 Hz)	OFHC	Grain size = 0.03 mm
Masumoto, (1944)	1.15				Alloying: Ni	Static, helical coils		
(1970)	1.11				Alloying: Ni	Vibrator-controlled oscillator (600-800 Hz)	99.972	ρ = 8.93 g/cm^3
Bundy,	1.28(d)				T = 20-300 K	Stress-strain initial slope	OFHC	Hard drawn
						Flexural resonance (~ 600 Hz)	99.999	*Modulus defect related to dislocation loop length, give frequency data where E ~ f^2
Mende, Biehl (1973)	1.19				Wire diameter 8-40 μm	Bending resonance		Average of 18 values

Table 9. Room-temperature elastic properties of polycrystalline copper (cont'd)

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
		10^{11} N/m^2 (except ν)						
Mercadier (1891)	1.207							
M'Farlane	1.15	0.434 0.464	(1.09)	(0.325)		Torsional pendulum		Annealed Quenched
Miller	1.27							
Morrow (1903)				0.33(d)		Mirror, lever apparatus		Drawn
Munse, Weil (1951)*	1.14				T = 77-477 K, specimen thickness, rolling direction	Stress-strain initial slope	OFHC	Annealed
	0.93(d) 1.16						Electrolytic tough pitch	Cold rolled 6% Annealed
	0.83(d) 1.17 0.94(d)						Deoxidized	Cold rolled 6% Annealed Cold rolled 6% *Young's modulus data for cold-rolled and annealed specimens seem to be interchanged in the original paper.
Nielsen (1968)	*				Low temperature (< 15 K) proton (8.9 MeV) bombardment, annealing	Resonant bar	99.999+	*Reports resonant frequency f, where E ~ f^2; reports modulus defect
Okuda (1963)	*				Plastic strain at 4 K, annealing temperature	Inverted torsion pendulum	99.999	Annealed 1 h at 800°C; *gives period of oscillation t, where E ~ t^{-2}
Okuda, Nakanii, Takamura, Maeta (1968)	*				Neutron irradiation (> 0.1 MeV), strain amplitude, annealing temperature	Transverse vibration (cantilever) resonance	99.999	
Orlov, Fedotov (1966)	1.16*	0.38*			Alloying: Ni			T = 673 K

Table 9. Room-temperature elastic properties of polycrystalline copper (cont'd)

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
		10^{11} N/m^2 (except ν)						
Pisati (1877), (1878), (1879)		0.389						
Reed, Mikesell (1967)	1.04	0.445	(0.53)	(0.17)	T = 4-295 K; alloying: Ni, Zn, others	Stress-strain initial slope	DHPC	Annealed
	1.30				T = 4-295 K		DHPC	Cold drawn 26%
Richards (1907)			1.85			Δv/v with respect to Hg		$B_{Cu}/B_{Fe} = 0.741$
Richards, Bartlett (1915)			1.32*			Glass piezometer	Commercial electrolytic	Hammered, turned; $B_{Cu}/B_{Fe} = 0.803$
Richards, Brink (1907)			1.96			Δℓ/ℓ		$B_{Cu}/B_{Fe} = 0.755$
Roth, Naundorf (1968), (1970)	rv				Isochronal annealing temperature after 3 MeV electron irradiation at 120 K	Flexural resonance (~ 2.5 kHz)	99.999	*B = 1.38 when corrected for iron standard according to Bridgman (1923)
Schaefer (1901)	0.971	0.389	(0.642)	0.245	T = 87-293 K	Bending interferometer	Chemically pure	
Schaefer (1902)		0.389						
Schneider, Burton (1949)	1.23	0.448	(1.61)	0.361		Rotating plate, ultrasound		
Schwinning, Strobel (1934)	1.14 1.20 0.71 1.09				T = 293-573 K	Stress-strain	Pure	Annealed (650°C, 1/2 h), static Hard drawn, static Annealed, dynamic Hard drawn, dynamic
Searle (pre-1914)	1.20							

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
		10^{11} N/m^2 (except ν)						
Shrivastava, Joshi (1972)	1.00	(0.421)	(0.532)	0.187		Flexural, torsional resonance		
Simpson, Sosin, Johnson (1972)	rv				Annealing temperature after electron irradiation		99.999	
Simpson, Sosin, Edwards, Seiffert (1971)	rv				Annealing time, annealing temperature after 1 MeV electron irradiation		99.9, 99.99	
Simpson, Sosin, Seiffert (1971)	rv				Annealing time, annealing temperature after 1 MeV electron irradiation	Resonant bar (~ 0.5 kHz)	99.999	
Smith (1953)	1.13				Plastic extension 1-8%, annealing 50-100°C	Free-free resonance	99.984 (OFHC)	Annealed 3 h at 500°C, 0.06 mm grain size
Soliman, Youssef, Essawi (1971)	0.64-0.93				Annealing temperature after drawing, plastic strain, grain size	Stress-strain initial slope	99.96 (Electrolytic)	Wires 0.56 mm diameter
Sosin (1962)	rv				Annealing temperature after electron (1 MeV) irradiation at 4 K	Resonant bar	99.999	
Sosin, Bienvenue (1960)	rv				Irradiation dosage (0.75 MeV), annealing temperature after irradiation	Cantilever resonance (400-600 Hz)	99.999	
Stokes (1960)	1.27				T = 73-773 K	Transverse vibration resonance	High conductivity	
Stromeyer (1894)	1.15	0.503	(1.08)	0.322		Interference apparatus		Rolled bar
	1.27	(0.463)	(1.65)	0.373				Cast

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Table 9.   Room-temperature elastic properties of polycrystalline copper (cont'd)

Reference	E	G	B	ν	Variables Studied	Method	Composition	Comments
		10^{11} N/m^2 (except ν)						
Subrahmanyam, Krisha Murty (1963)	1.232	0.449	(1.603)	(0.372)	Plate thickness	Ultrasoric wedge		0.9 mm plate
Subrahmanyam, Krishnamurty (1963)	1.232	0.449	(1.603)	(0.372)	Alloying: Ag, Au, Ni, Pb	Ultrasoric wedge		0.94 mm plate
Sutherland (1891)	1.351				T = 389-375 K			ρ = 8.95 g/cm^3
Talashkevich, Kostin, Aleksandrov (1964)	1.220	0.459	(1.189)	(0.329)		Dynamic	M1 Copper	Wire ~ 7 mm diameter, consider texture problem
	1.203	0.451	(1.206)	(0.334)			M3 Copper	
Tomlinson (1886)	1.290	0.439	(1.049)	(0.469)				
Townsend, DiCarlo, Nielsen, Stabell (1969)	*				Irradiation flux of 10 MeV protons at T < 15 K		99.999	Wire 0.013 cm dia., vac. annealed 600-900°C, RRR = 1000, *relative frequency
van den Beukel, Brouwer (1968)		rv			Plastic torsion strain to 18%, alloying: Au	Torsional vibrations	99.999	Plastically strained torsionally 0.19%
van den Beukel, Deelen (1967)		rv			Vibration amplitude	Torsional vibrations		
Voigt (1884)				~ 0.25				Electrolytically deposited
Voigt (1893)	1.064						Impure cast plate	
Wawra (1967)	1.326	0.460	(3.76)	(0.441)	Annealing treatment, alloying: Zn	Longitudinal, torsional resonance	Electrolytic	Annealed at 500°C 3 h
Wawra (1968)	1.28	0.460	(1.96)	(0.391)	Annealing treatment, alloying: Zn	Longitudinal, torsional resonance		Recrystallized
Wenzl, Kerscher, Fischer, Ehrensperger, Papathanassopoulos (1971)		rv			Neutron irradiation at 4 K	Helical spring vibration (50-300 Hz)	99.999	Annealed 2 h at 600°C

Table 9. Room-temperature elastic properties of polycrystalline copper (cont'd)

Reference	E	G 10^{11} N/m^2	B (except ν)	ν	Variables Studied	Method	Composition	Comments
Wertheim (1848)	1.140							
Youngs (1958)	1.20	0.46	(1.02)	0.303		Optical diffraction grating		
Zakharov (1962)	1.288				Neutron irradiation, vibration amplitude	Flexural vibrations	99.95 (Electrolytic)	Annealed 600°C 2 h
Zetzsche, Hauser (1962)	rv				Irradiation		99.985	
Average values	1.235	0.454	1.402	0.350				
Standard errors	0.007	0.012	0.039	0.009				

Table 10. Summary of average values and standard errors of elastic constants of copper at room temperature; all entries, except ν, have units of 10^{11} N/m^2.

Polycrystal

$E = 1.235 \pm 0.007$
$G = 0.454 \pm 0.012$
$B = 1.402 \pm 0.039$
$\nu = 0.350 \pm 0.009$

Single-crystal

$C_{11} = 1.691 \pm 0.002$
$C_{12} = 1.222 \pm 0.003$
$C_{44} = 0.7542 \pm 0.0005$
$C' = 0.2346 \pm 0.0005$
$B = 1.376 \pm 0.002$
$C_L = 2.211 \pm 0.003$
$C_{111} = -13.5 \pm 0.3$
$C_{112} = -8.0 \pm 0.3$
$C_{123} = -1.2 \pm 0.3$
$C_{144} = 0.66 \pm 0.7$
$C_{166} = -7.2 \pm 0.3$
$C_{456} = -0.32 \pm 0.4$

Polycrystal derived from single-crystal data

VRH-Arithmetic		Ledbetter, Naimon		
$E =$	1.273	1.222	$\pm$	0.003
$G =$	0.473	0.452	$\pm$	0.001
$B =$	1.378	1.378	$\pm$	0.002
$\nu =$	0.346	0.352	$\pm$	0.004

Table 11. Temperature coefficients of the second-order elastic constants of copper at room temperature; entries have units of 10^{-4} K^{-1}.
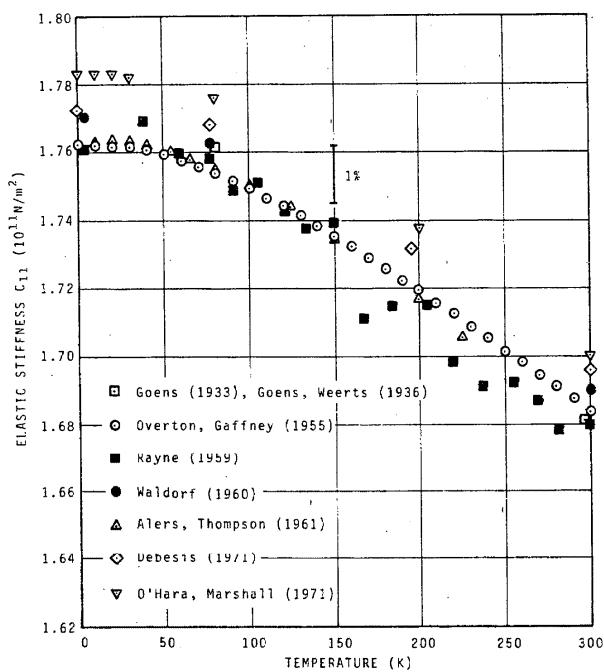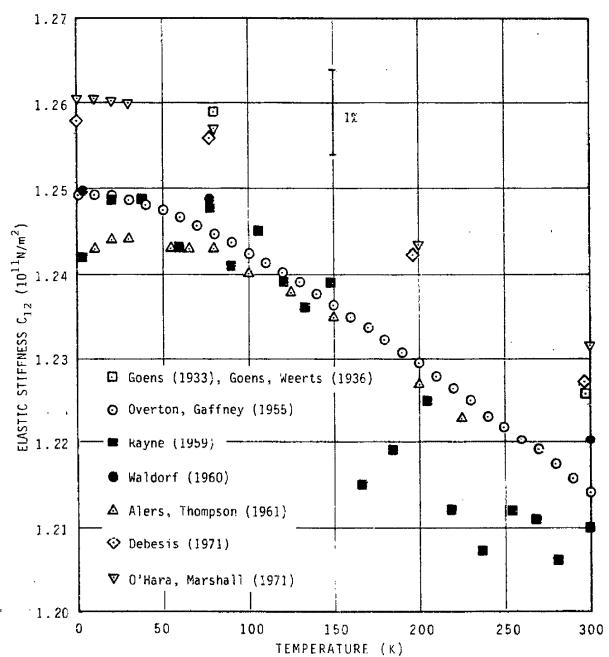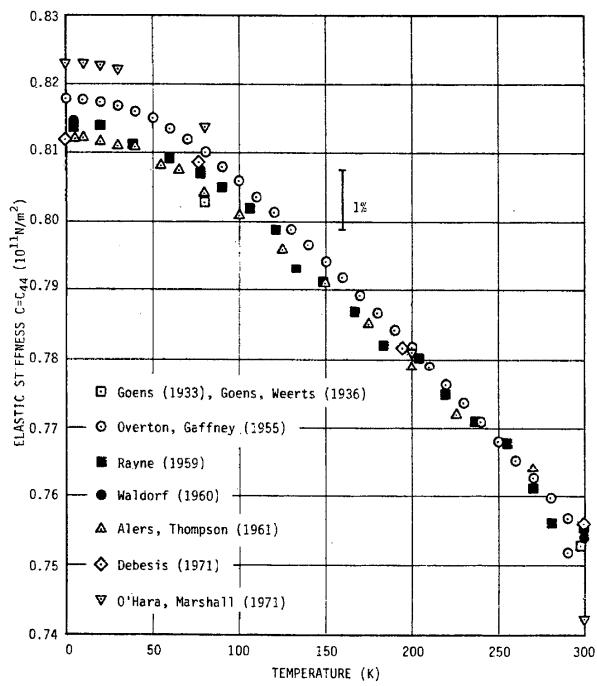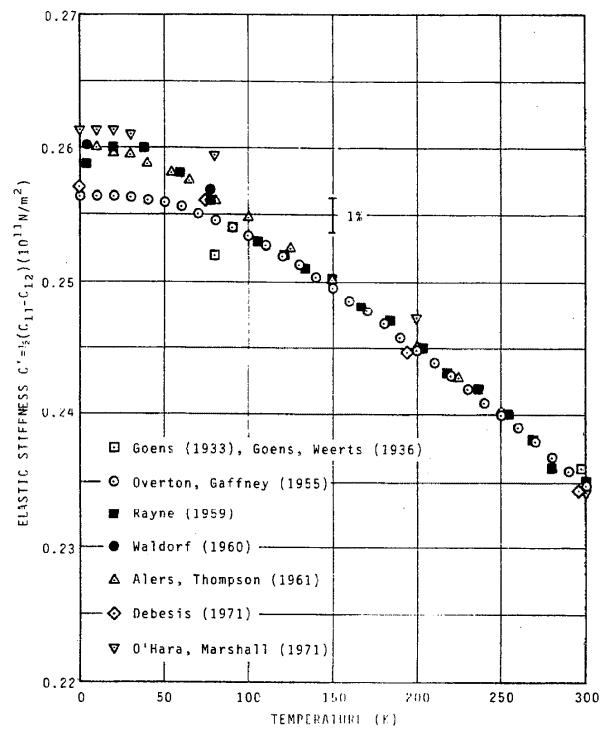
| Reference	$\frac{1}{C_{44}}\frac{dC_{44}}{dT}$	$\frac{1}{C'}\frac{dC'}{dT}$	$\frac{1}{C_{11}}\frac{dC_{11}}{dT}$	$\frac{1}{C_L}\frac{dC_L}{dT}$	$\frac{1}{C_{12}}\frac{dC_{12}}{dT}$	$\frac{1}{B}\frac{dB}{dT}$	Comments
Goens (1933); Goens, Weerts (1936)	-3.0	-3.1	-1.8	-2.1	-1.2	-1.5	Room-temperature and 80 K values only
Overton, Gaffney (1955)	-3.7	-4.3	-2.1	-2.4	-1.2	-1.6	4.2 - 300 K data
Rayne (1959)	-3.2	-4.1	-2.0	-2.2	-1.1	-1.5	4.2 - 300 K data
Waldorf (1960)	-3.3	-4.1	-2.1	-2.3	-1.4	-1.7	4.2 - 300 K data
Alers, Thompson (1961)	-3.4	-4.2	-1.9	-2.1	-1.0	-1.3	4.2 - 250 K data
Chang, Himmel (1966)	-3.6	-4.4	-2.4	-2.6	-1.6	-1.9	300 - 800 K data
Debesis (1971)	-3.4	-4.4	-2.1	-2.3	-1.2	-1.5	297, 195, and 77 K values only
O'hara, Marshall (1971)	-4.0	-4.6	-2.1	-2.5	-1.1	-1.5	4.2 - 300 K data; 0.23 at. % Au
Average values	-3.5	-4.2	-2.1	-2.3	-1.2	-1.6	
Standard errors	0.1	0.2	0.1	0.1	0.1	0.1	

Table 12. Temperature Coefficients, at Room Temperature, of the Polycrystalline elastic constants of copper; entries have units of 10^{-4} K^{-1}.

	$\dfrac{1}{E}\dfrac{dE}{dT}$	$\dfrac{1}{G}\dfrac{dG}{dT}$	$\dfrac{1}{B}\dfrac{dB}{dT}$	$\dfrac{1}{\nu}\dfrac{d\nu}{dT}$
Polycrystal data	-4.8	-7.1		
Averaged single-crystal data	-3.6	-3.9	-1.6	0.8

Table 13. Summary of studies on the effects of alloying on the polycrystalline elastic constants of copper.

	Parameter				
Reference	E	G	B	ν	Alloy Element(s)
Alers, Liu (1967)	x				Zn
Cabarat, Guillet, LeRoux (1949)	x				Sn, Zn
Druyvesteyn, Meyering (1941)	x	x	x	x	Zn
Faninger (1969)	x	x	(x)	x	Ni
Fukuroi, Shibuya (1950)	x				Ni
Greer, Bucknall (1964)	x				Ni. Zn
Guillet (1939), (1940)	x				Al, Sn, Zn
Hopkin, Pursey, Markham (1970)	x	x	x	x	Al, As, Au, Ga, Ge, In, Mn, Ni, Si, Sn, Zn
Köster, Rauscher (1948)	x				Au, Ni, Pd, Pt
Lenkkeri, Lähteenkorva (1973)	x	x	x	(x)	Al
Liu, Alers (1966)	x				Zn
Masumoto, Saito (1944)	x				Ni
Masumoto, Saito, Sawaya (1970)	x				Ni
Orlov, Fedotov (1966)	x	x			Ni
Reed, Mikesell (1967)	x	x	(x)	(x)	Ni & others, Zn & others
Subrahmanyam, Krishnamurty (1963)	x	x	(x)	(x)	Ag, Au, Ni, Pb
Wawra (1967)	x	x	(x)	(x)	Zn
Wawra (1968)	x	x	(x)	(x)	Zn

FIGURE 1. Temperature variation of C_{11} of copper.

FIGURE 2. Temperature variation of C_{12} of copper.

FIGURE 3. Temperature variation of C_{44} of copper.

FIGURE 4. Temperature variation of $C' = \frac{1}{2}(C_{11} - C_{12})$ of copper.
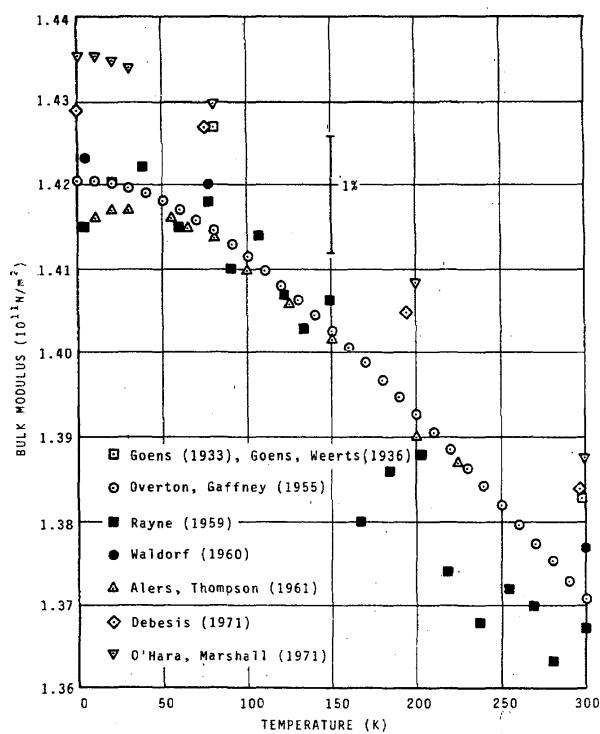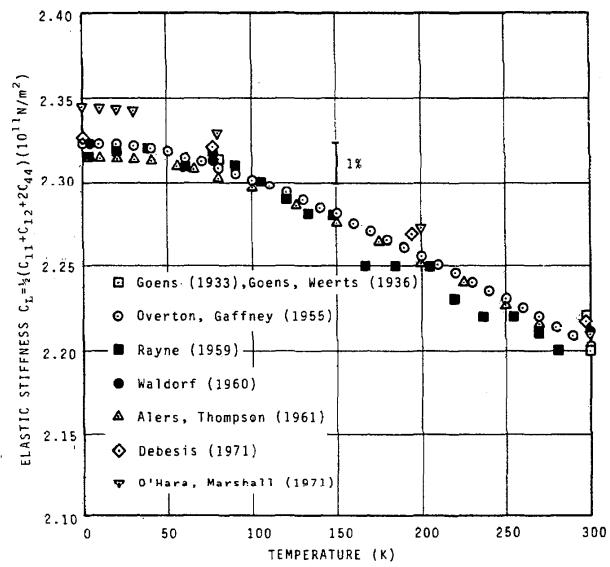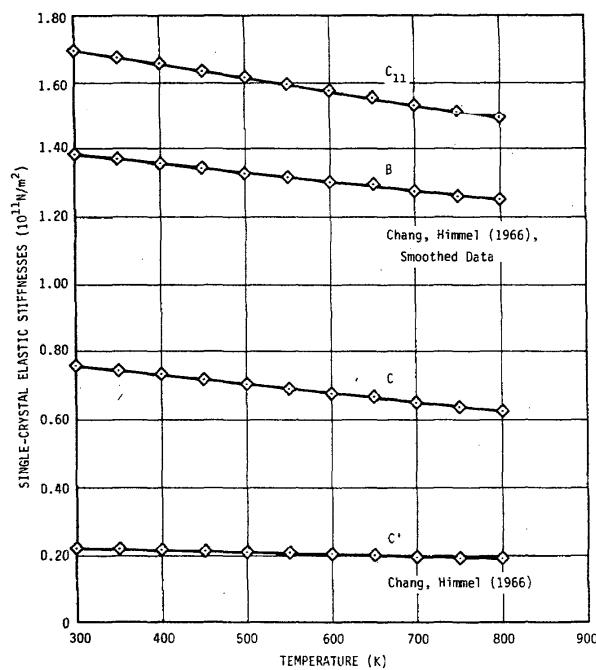
FIGURE 5. Temperature variation of $B=\frac{1}{3}(C_{11}+2C_{12})$ of copper.

FIGURE 6. Temperature variation of $C_L=\frac{1}{2}(C_{11}+C_{12}+2C_{44})$ of copper.

FIGURE 7. High–temperature variation of single–crystal elastic constants of copper.
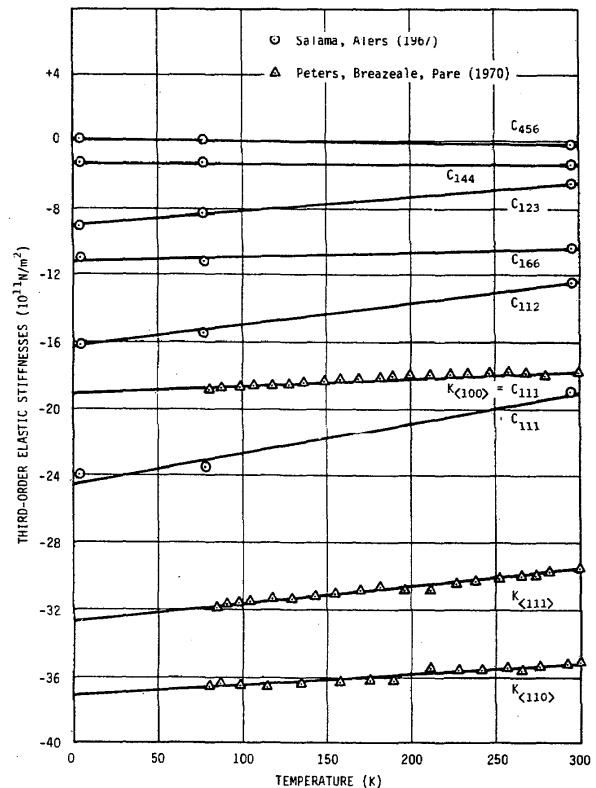
FIGURE 8. Temperature variation of third-order single-crystal elastic constants of copper.
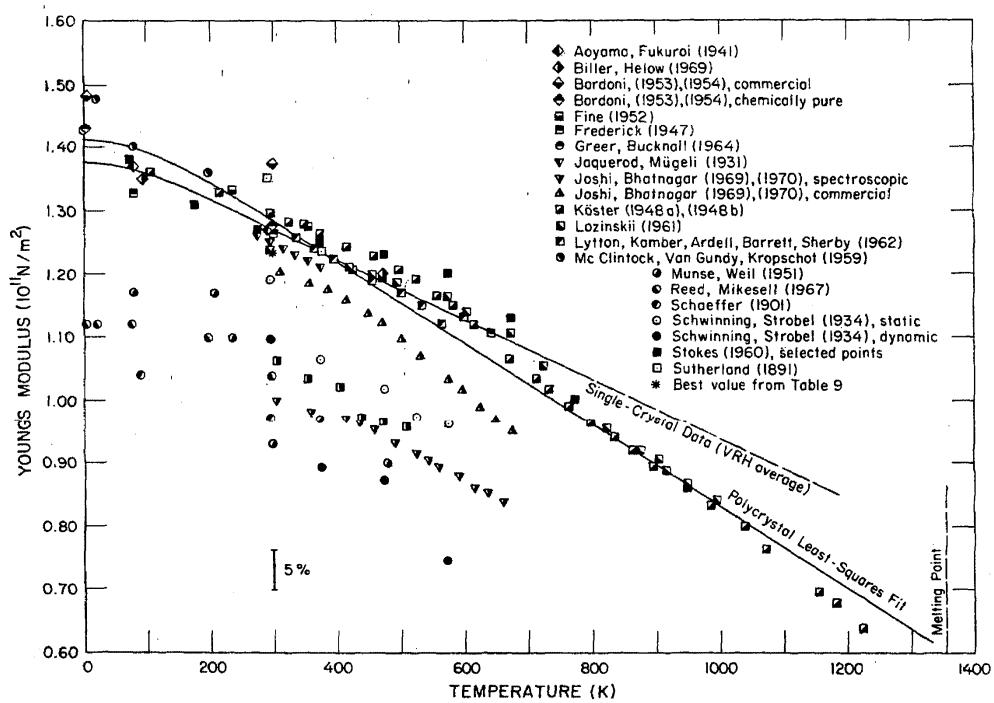
FIGURE 9. Temperature variation of Young's modulus of copper.
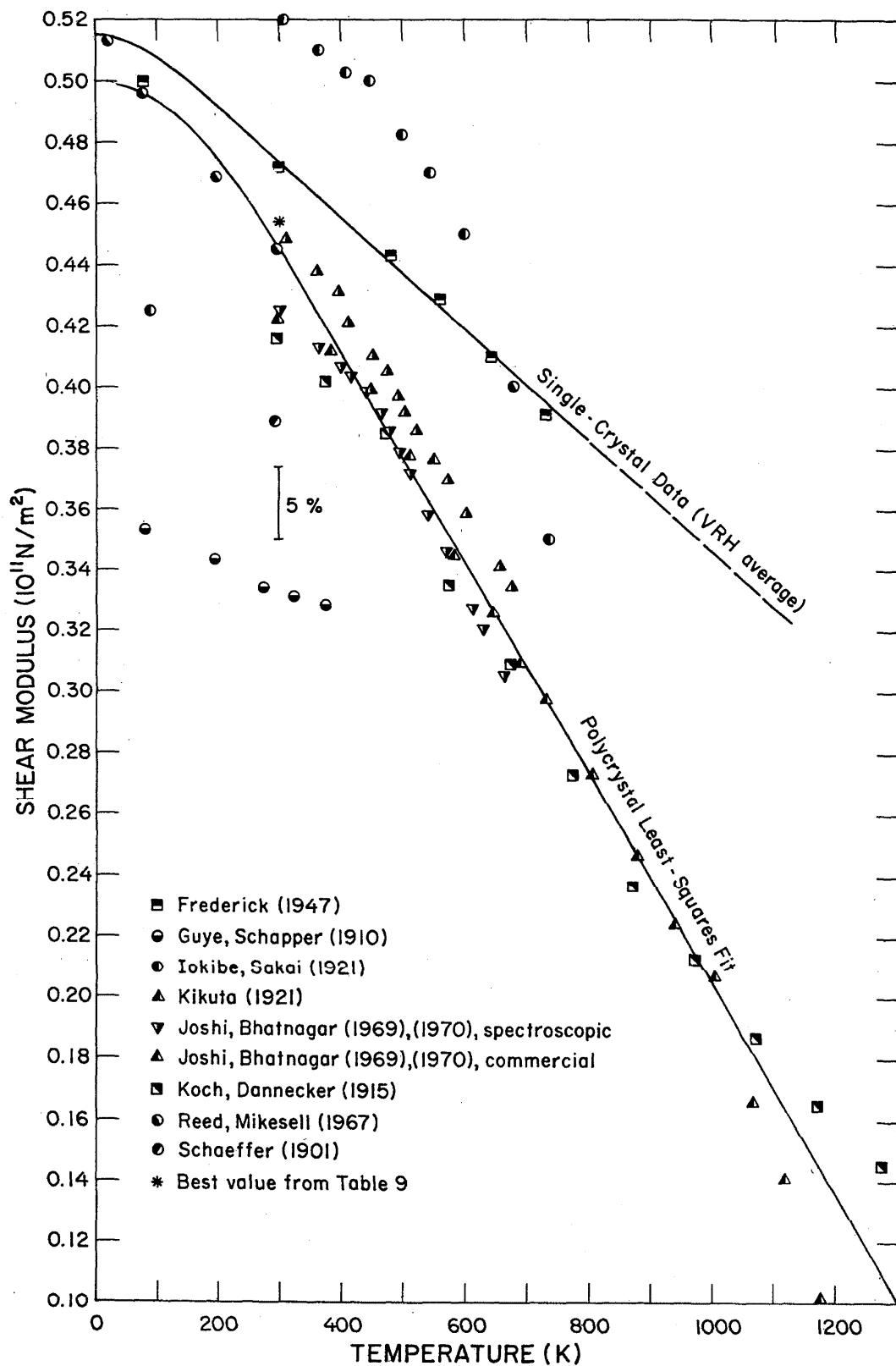
FIGURE 10. Temperature variation of shear modulus of copper.
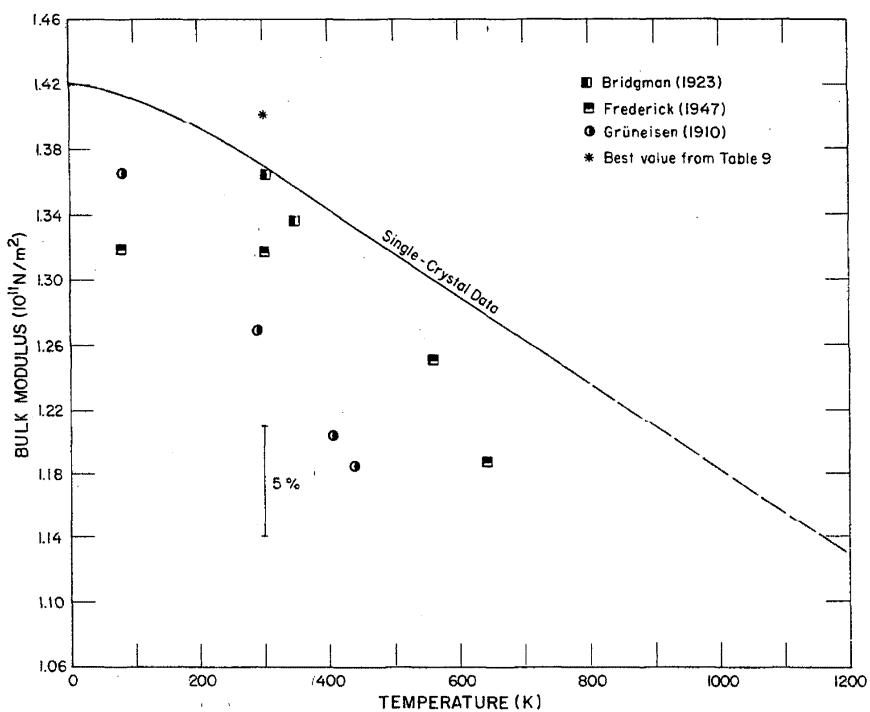
FIGURE 11. Temperature variation of bulk modulus (reciprocal compressibility) of copper.
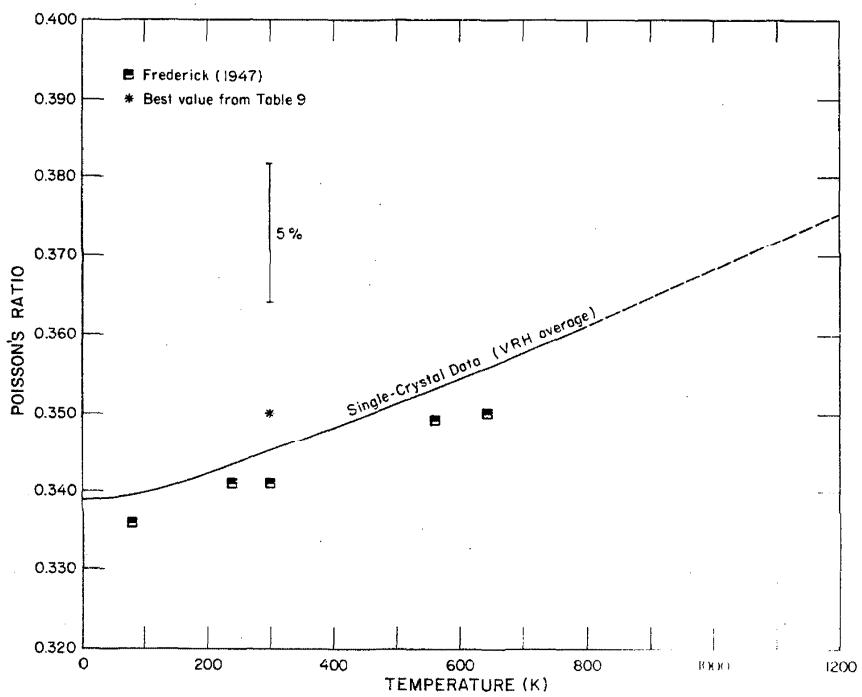

FIGURE 12. Temperature variation of Poisson's ratio of copper

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