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Savely G. Karshenboim, Peter J. Mohr, and David B. Newell

Advances in Determination of Fundamental Constants

Savely G. Karshenboim[a)]
Max-Planck-Institut für Quantenoptik, Garching 85748, Germany and Pulkovo Observatory, Saint Petersburg 196140, Russia

Peter J. Mohr[b)] **and David B. Newell**[c)]
National Institute of Standards and Technology, Gaithersburg, Maryland 20899-8420, USA

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We present a brief overview of the presentations at the workshop on the determination of the fundamental constants (Eltville, 2015) and the contributions to the proceedings. © *2015 AIP Publishing LLC.* [http://dx.doi.org/10.1063/1.4926575]

Key words: fundamental constants; International System of Units; natural units; precision physics.

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1. Introduction

The fundamental physical constants reflect properties of the most important objects and phenomena in our world, including the size and mass of very large objects like the stars and very small objects like the atoms. One has to deal with the fundamental constants while working on fundamental problems like the creation and evolution of the universe. Many devices (such as clocks and electronics) and many services (such as Global Positioning System navigation), which are important for our practical life, need very advanced technologies and are based on high-precision standards, which in turn are based on fundamental constants and natural phenomena.

Fundamental constants set the most stable references existing in the world, and we need to relate them to our standards. Being universal properties, many fundamental constants can be accessed through very different phenomena. The data, which determine the value of the constants of nature, are diverse. They are also correlated, because in many experiments, we use the same standards directly or indirectly and the same assumptions.

Many of the determinations of the fundamental constants are of practical interest. The related work involves current and perspective measurement standards in one or another way. It also deals with a need for measurements in “exotic areas,” such as weak macroscopic forces involved in the determination of the Newtonian constant of gravitation. The procedure to elaborate the most accurate values of such “practical” fundamental constants is the least-square adjustment (LSA) process. It is regularly performed by the CODATA Task Group on Fundamental Constants.[1] The former evaluations were done during a period of over 50 yr.[2–7] The current evaluation has produced the set of CODATA Recommended Values of the Fundamental Constants: 2014, which includes the data available by December 31, 2014. The numerical results are given at the website of the National Institute of Science and Technology (NIST),[8] with details to be explained in a full paper within a year.

Here, we present proceedings of the workshop on the Determination of the Fundamental Constants. The workshop took place on February 1–6, 2015 at the Hotel Frankenbach, Eltville, Germany;[9] 78 participants from 11 countries registered and participated in the workshop and delivered 46 talks, 4 brief communications, and 21 posters.

The presentations at the workshop cover a broad range of phenomena. The fundamental constants play a key role in

[a)]Electronic mail: savely.karshenboim@mpq.mpg.de.
[b)]Electronic mail: mohr@nist.gov.
[c)]Electronic mail: dnewell@nist.gov.

microscopic physics. Simple atoms are described by quantum electrodynamics (QED), which requires numerical values of a few fundamental constants, such as masses of the electron and other relevant particles and the fine-structure constant, as the input parameters. The QED theory of simple atoms is well represented in the contributions to this volume. More complex atoms cannot be described as simply, and their description involves phenomenological parameters such as nuclear radii and magnetic moments. These parameters are partly found using the LSA process and partly by complementary evaluations. Such evaluations are presented in a few contributions. Examples of fundamental constants found by means of microscopic physics are the Rydberg constant R_∞, the fine structure constant α, and the electron and proton masses expressed in atomic mass units.

Some of the experiments involve macroscopic physics with various balances, etc. Such experiments based on classical physics have their limitations. The properties of macroscopic bodies are not universal and may vary over on time. Hence, the main trend in building measurement standards is to turn to quantum phenomena, some of which are based on the constants of nature.

There are two "intermediate" groups of experiments. One group is based on thermodynamics and statistical physics. In principle, any thermodynamic property of a substance is determined by quantum properties of its atomic and molecular constituents and might be expressed in such terms. However, a particular sample has certain unique properties due to its chemical and isotopic composition, non-ideality of the crystal structure, etc. Two examples of "thermodynamic" constants are the Boltzmann constant k and the Avogadro constant N_A. In both cases, the accurate determination of the isotopic composition of the substance used for the measurement plays a crucial role and is one of the largest sources of uncertainty.

The other group of experiments, which combine microscopic and macroscopic physics, rely on macroscopic quantum effects. The quantum Hall effect and Josephson effect give measurement results in terms of fundamental constants, in particular, the Planck constant h and the elementary charge e.

This workshop was designed to bring together experts from the various fields involved in determining values of the fundamental constants. Although the data were available by the end of 2014, the discussions of the results in early 2015 serve to facilitate the evaluation of the data. Below, we give a brief overview of the presentations made at the workshop and the written contributions to the proceedings volume.

2. Overview of the Data

2.1. Rydberg constant and proton radius: R_∞ and r_p

The LSA deals with various sets of diverse data with very different levels of accuracy. One of the most accurately known fundamental constants is the Rydberg constant,

$$R_\infty = \frac{\alpha^2 m_e c}{2h}, \tag{1}$$

determined by means of the spectroscopy of atomic hydrogen and deuterium. Most of the important results have been obtained more than a decade ago and have been well reviewed in Refs. 4–7. One of the projects under development now is an experiment at the Laboratoire Kastler Brossel (LKB) on the $1s-3s$ transition in hydrogen reported in this proceedings by Galtier *et al.*[10] There are other promising projects that exist, but they have not yet reached the stage where preliminary results are available.

The determination of the Rydberg constant by spectroscopic measurements requires measurement of at least two transitions in order to determine two values, one for the Rydberg constant itself and one for the proton charge radius r_p. Another possibility for determination of R_∞ is to use the combination of a value from a frequency of a hydrogen transition, which usually is the $1s-2s$ transition,[11] which is the most accurately known, and a value of the proton charge radius determined another way. There are two such independent methods of determining r_p, both of which are presented in this proceedings.

The result with the best claimed accuracy has been obtained in a study of the Lamb shift in muonic hydrogen. The muon is a particle which lives only 2 μs; however, this is sufficient for a precision experiment to be carried out. Such a measurement was performed at the Paul Scherrer Institut (PSI) by an international collaboration.[12] The effect of the nuclear structure on the transition frequency is enhanced and the required accuracy for a determination of the proton radius is substantially lower than for ordinary hydrogen. That is true for both experimental and theoretical accuracies. The theory of the Lamb shift in muonic hydrogen is much simpler than that for ordinary hydrogen; the up-to-date theory of the Lamb shift in muonic hydrogen is reviewed in this proceedings by Karshenboim *et al.*[13]

There is also a nonspectroscopic method to determine the value of the proton charge radius based on study of elastic electron–proton scattering as reported in this proceedings by Arrington.[14] To find the radius, one has first to measure the e–p cross section at various values of the momentum transfer and then analyze the results in terms of the electric and magnetic form factors of the proton. The electric form factor $G_E(q^2)$ is related to the charge-distribution density. The Fourier transform of the charge density is related to the electric form factor at low momentum transfer. The proton charge radius is the slope of the from factor $G_E(q^2)$ at zero momentum transfer. Unfortunately, it is not possible to measure the form factor directly at $q^2 = 0$, so the value of the radius is obtained by fitting the data and extrapolating to $q^2 = 0$ [see Arrington[14] and Arrington and Sick[15] in this proceedings for detail]. In principle, different extrapolations are possible based on the same data. A summary of the determination of the proton charge radius from the scattering data is presented in Ref. 15.

Unfortunately, the data on the proton radius are not consistent; the muonic hydrogen value strongly disagrees with the others. Meanwhile, the hydrogen-spectroscopy result is in perfect agreement with the results of the phenomenological evaluation of scattering data. We expect that more experimental and theoretical results will appear and help resolve the controversy.

2.2. Electron to proton mass ratio: m_e/m_p

Another group of data which have a comparable accuracy are from determination of the mass ratios for various atoms and nuclei. The bulk evaluation of this information is performed by the Atomic Mass Data Center (AMDC).[16] The procedure of the atomic mass evaluation is performed regularly; however, its schedule is not compatible with the regularity of CODATA's LSA, so some mass ratios and the related correlations are taken from the original works and included in the LSA.

There are also a few exceptions. The masses of atoms and/or nuclei of certain hydrogen and helium isotopes play an important role in LSA, and the relevant ratios are always studied within LSA. An even more important case is the evaluation of m_e/m_p. In previous evaluations (see, e.g., Refs. 4–7), the determination of the electron-to-proton mass ratio and the electron mass in relative atomic mass units was essentially less accurate than that of the proton mass in those units and other atomic mass ratios. In other words, the determination of m_e/m_p, which was somewhat less accurate, was separate from the atomic mass evaluation procedure.

In a few recent LSAs, the dominant method of determining the electron-to-proton mass ratio was to study the g factor of a bound electron in hydrogen-like carbon-12 or oxygen-16, as done at Univertität Mainz. That provided the electron mass in atomic mass units. Together with the proton mass, even more accurately known in atomic mass units, one can obtain a value for m_e/m_p. Recently, with the improvement of the accuracy of the Mainz experiment[17] (for a theoretical review see Shabaev *et al.* in this proceedings[18]), the accuracy of the determination of the electron mass in relative atomic mass units has become compatible with the accuracy of the determination of the proton mass in those units.[8]

If a direct determination of the ratio m_e/m_p were also improved, that would make the structure of the correlations of the mass-ratio-related data more complicated.

2.3. Fine structure constant: α

The determination of the electron-to-proton mass ratio is also related to a determination of the fine-structure constant,

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c}, \tag{2}$$

which plays a key role in calculations of numerical values of theoretical predictions in atomic physics and in electromagnetic phenomena. Progress in its determination is due to improvement in both experiment[19] and theory[20] of the anomalous magnetic moment of the electron.

To produce a reliable result for α, it is best to have an independent accurate determination. Currently, the second most accurate determination is from recoil spectroscopy,[21] which is applied to measure mc^2/h for rubidium and cesium atoms. Combining this result with a value of Rydberg constant (1), the mass of the atoms of interest in atomic mass units, and the electron mass in the atomic units, one can find α. The recoil spectroscopy result creates a correlation between measurements of the electron mass in atomic mass units and α.

To obtain a value of α from the anomalous magnetic moment of the electron, one has to deal with the most advanced QED theory for free particles. At present, the theoretical expression for the anomalous magnetic moment of electron[19] includes five-loop contributions. To verify the calculations, by comparing them with an experimental value, one has to take a certain value of α. The recoil value supplies us with a possibility to perform the most accurate test of QED with the anomalous magnetic moment of electron.

In principle, there are many other possibilities to determine α. They are not as accurate as two mentioned above, but they are very important because they are based on different physics. One possibility is for an "electric" determination of α based on the quantum Hall effect, which is very important for metrology. Another one, which deserves to be mentioned, is based on the calculation and measurement (see Marsman *et al.* in this proceedings[22]) of the $2P$ fine-structure interval in neutral helium. An exmaple of a theory for the helium atom can be found in Yerokhin and Pachucki in this proceedings,[23] where another transition (2S-2P) has been investigated.

2.4. Planck and Avogadro constants: h and N_A

While R_∞, m_e/m_p, and α are examples of microscopic-physics constants, there are also fundamental parameters that are determined through macroscopic experiments. Macroscopic physics provides the determination of a few important constants, and three constants determined with the highest accuracy for such classical experiments are the Planck constant h, the elementary charge e, and the Avogadro constant N_A. They are constants from three rather different areas of physics. The Planck constant marks quantum phenomena, the elementary charge plays a crucial role in any description of electromagnetic phenomena, and the Avogadro constant is a key parameter for thermodynamics. Still, the determination of their numerical values is closely related.

From the point of view of the most precise determinations, the Planck constant is an "electric" constant. This is due to macroscopic quantum phenomena, such as the Josephson effect and the quantum Hall effect. The latter allows us to determine the von Klitzing constant,

$$R_K = \frac{h}{e^2} = \frac{1}{2\alpha\epsilon_0 c},$$

with high accuracy. We know the numerical value of R_K as well as we know α. This allows one to express any result for h in terms of e and vice versa.

Similarly, the Josephson effect involves the Josephson constant in the determination of h through the equation

$$K_J = \frac{2e}{h}.$$

We do not know the numerical value of the Josephson constant with high accuracy, which is why the experiments are organized in such a way that one works with voltages calibrated in terms of K_J (and a measured frequency) and resistances measured in units of R_K. The most successful experiments where such a scheme was realized are with watt balances.[24–26]

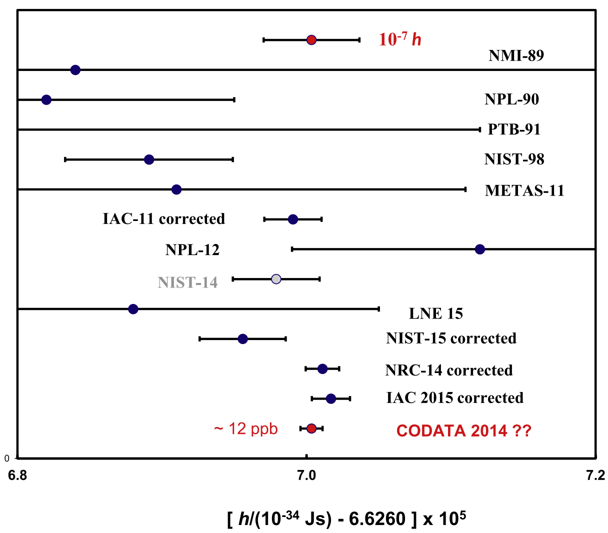

FIG. 1. Determination of the Planck constant as presented in the talk by Barry Wood (NRC). Courtesy of Barry Wood.

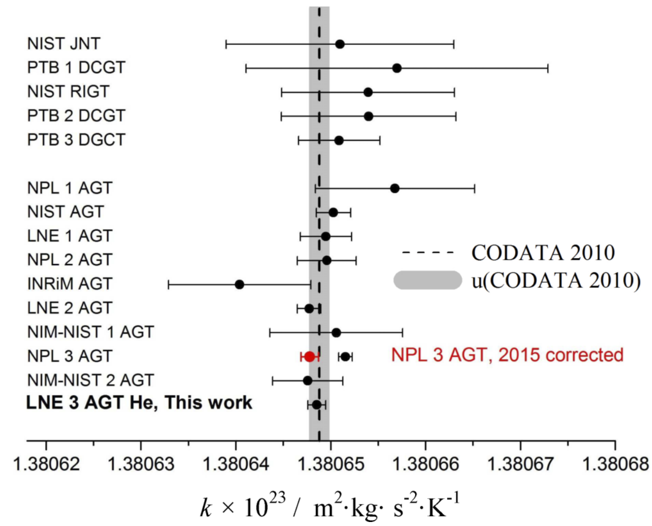

FIG. 2. Determination of the Boltzmann constant as presented in the talk by Laurent Pitre (LNE). Courtesy of Laurent Pitre.

They determine a value of the Planck constant at the level of a few parts in 10^8. Unfortunately, the scatter of the results is slightly larger than the uncertainty. The summary of the situation is presented in Fig. 1, where the most accurate watt-balance results are marked as NIST-98,[24] National Research Council (Canada) (NRC)-14,[25] and NIST-15.[26] For more details, see a recent topical issue of Metrologia on the watt balances.[27]

One may also determine the value of the Planck constant using a completely different approach. It is based on the fact that the molar Planck constant hN_A is known to be very accurate. That results from the high accuracy of the determination of various microscopic masses in relative atomic mass units and in frequency units (i.e., by measuring the related value of mc^2/h instead of a mass m). Both measurements can be made much more accurately than a fractional uncertainty of 10^{-8}. Recalling that the numerical value of the Avogadro constant is by definition a conversion factor from the kilogram to the relative atomic mass unit, we conclude that hN_A serves as the conversion factor between the mass measurements in atomic mass units and frequency units. Since we know the masses in those units accurately, we also know the conversion factor. This allows one to transform a determination of the Avogadro constant into a determination of the Planck constant (see Massa *et al.*[28] and Borys *et al.*[29] in these proceedings). The N_A-induced result for h is presented in Fig. 1 as The International Avogadro Coordination project 2015.

Historically, the determination of the Avogadro constant was based on natural silicon and its accuracy suffered from uncertainty in determination of the isotopic composition. That problem has been resolved by enriching the silicon, which means a drastic reduction of the abundance of the heavier isotopes of silicon. The first result with the enriched silicon crystal has the fractional uncertainty of 3 parts in 10^8.

When the first result with the enriched silicon crystal appeared, it did not agree with the most recent watt-balance experiment result.[30] The situation was seen as a contradiction between an "electric" h-experiment and a "material" N_A experiment. Currently with two accurate watt-balance results, we see some difference between them, while the N_A experiment agrees well with one of them.[25]

2.5. Boltzmann constant: k

The determination of another thermodynamic constant, the Boltzmann constant, is somewhat isolated from most of the other values involved in the CODATA LSA evaluation. It can be measured by a few methods, but presently only acoustic gas thermometry (AGT) can produce a result with uncertainty at the level of 1 ppm. Still, experiments with other methods are in progress. Unfortunately, at this moment, we see not only the appearance of few independent AGT results but also a contradiction between two of the most accurate ones, namely, between the Laboratoire National d'Essais (LNE)[31] and the National Physical Laboratory (NPL)[32] (see Fig. 2).

The discussions at this workshop provide some hope that the controversy between the NPL and LNE results will be discovered soon. It appears that a more accurate account of the isotopic abundance of argon isotopes will move the NPL result toward the LNE value.[33] For more details, see a topical issue of Metrologia on the Boltzmann constant,[34] which is scheduled for July, 2015.

2.6. Newtonian gravitational constant: G

The Newtonian constant of gravitation G is independent of the other constants. While others involved in the LSA play a role in contemporary or prospective standards, the "big G" value is purely of physical interest; it definitely is one of the most fundamental constants. Its order of magnitude, together with h and c, determines the so-called Planck scale, the scale of the masses, time intervals, and distances where the fundamental laws of nature are quite different from those we deal within our laboratories or in our observation of astronomical objects.

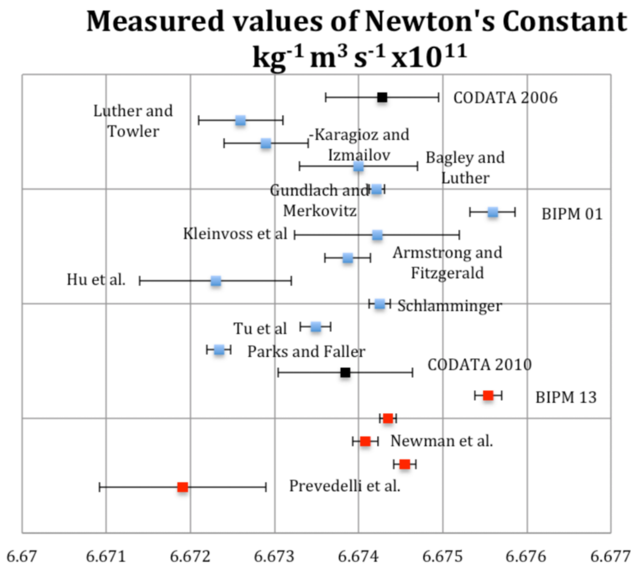

FIG. 3. Determination of the Newtonian constant as presented in the talk by Clive Speake (University of Birmingham). Courtesy of Clive Speake.

However, the exact numerical value of G tells us more about the international prototype of the kilogram, stored at the Bureau international des poids et mesures (BIPM) in Sévre, than about gravity. Gravity plays a somewhat marginal role in the interaction of two laboratory scale macroscopic bodies. To determine the gravitational interaction between such bodies, one has to deal with weak, or rather with "ultraweak," forces and to control various small deformations, effect of scattered charges, and other competitive "marginal effects." Because of such an environment, the very determination of the numerical value of G is a challenging problem of classical metrology. The accuracy of separate determinations has reached the level of few parts per million. However, the scatter of the data does not allow the fractional uncertainty of the recommended value to be far below the 10^{-4} level (see Fig. 3). The situation is well reviewed by Quinn and Speake,[35] where the original references to the experiments can be found.

Still, there are a number of gravitational effects studied with extremely high accuracy, which sometimes reaches a level of uncertainty much below a part per billion. In particular, that happens in the study of celestial mechanics within our solar system. The constant of gravity, which plays a role there, is a product of the standard gravitational constant G and the solar mass $M_\odot$. The value of $GM_\odot$ is known as the heliocentric constant of gravity and its determination is considered in Ref. 36. Sometimes, the heliocentric constant of gravitation is understood as G measured in astronomical units. While the astronomical units of the time and length are simply expressed in terms of the related SI units with an appropriate accuracy, the mass unit used is different since the astronomical mass unit is the solar mass.

2.7. Other data

Concluding the overview of the data, we mention two important issues.

First, one can note that QED plays an important role in the determination of various "microscopic" constants (see, e.g., reviews[13,18,23] on particular QED systems). It is important to consider that there are different sectors of QED. Some values are linked to QED of free particles and some deal with bound state-QED. Bound-state QED is applied to two-body or three-body systems.

It may happen that QED is used in different ways to determine the same value, but it is often not the same sector of QED. For instance, m_e/m_p can be found from the g factor of a bound electron and from spectroscopy of the antiprotonic helium atom, which both need QED. However, the bound-g theory deals with a two-body system with an enhanced Coulomb interaction (with $Z = 6, 8$) and a quite relativistic electron[18] ($v/c \simeq Z\alpha$), while in antiprotonic helium, a standard Coulomb interaction is used (the effective value of Z is about two for the antiproton and unity for the electron) and there are relatively small relativistic effects for this three-body system.

We rely on QED, and we have to check whether our approaches to particular systems are appropriate. In this respect, the area of "QED tests" (see, e.g. reviews 38 and 39) is closely related to the determination of atomic constants. Some tests of QED do not result in a very accurate constant, but still are interesting as a test of the approaches used for values of other constants obtained in the LSA. The study of the anomalous magnetic moment of the muon (see, e.g., Ref. 37) plays marginal role in the LSA; however, it tests the theory of the anomalous magnetic moment of the electron, which is crucial in the determination of α. Studies of positronium are in general important to check our proficiency in estimating various recoil effects, and the new precise measurement of the hyperfine interval in positronium[40] contributes to such understanding.

The other issue, worth mentioning, is that the LSA is not the only evaluation that deals with bulk data on various fundamental parameters. There are a few others, focused on certain groups of data. We have already mentioned the atomic mass evaluation.[16] There are many evaluations of nuclear radii[41,42] and nuclear magnetic and quadrupole moments.[43] Some of these can contribute to the LSA when more spectroscopic data become available (see, e.g., Ref. 41), while others are important for nuclear physics.[42,43]

3. Summary

The workshop included the presentation of a number of new results and has allowed them to be discussed in detail. Improvement in accuracy was achieved in determinations of α, m_e/m_p, and k. We have learned more about reliability of the results on h. The situation with R_∞ and G remains to a certain extent uncertain. It is important that k and h are among the constant for which apparent progress has taken place. These two constants are very important for the redefinition of the SI system in terms of fundamental constants.

It is interesting to note that the early metric system was based on natural quantities, such the size of earth, the density of the water, and the astronomical year. Later on, it was discovered that the access to such quantities is problematic and the standards became defined in terms of certain artificial bodies. One of them, the international prototype of the kilogram, is still a cornerstone of the whole International System of Units, SI,[44] since the ampere, the mole, and the candela are linked to it. Meanwhile, the kelvin is still based on a property of a certain "standard" water.

The definitions of units, which look very general, are based in fact on our ability to build a system of efficient standards using these definitions. It seems that we have approached the moment when the use of standards based on natural constants and quantum phenomena is more advantageous than the use of those based on the traditional definitions. The physical units of the new "quantum" SI system are to be based on certain fixed adopted values of the hyperfine splitting in cesium, speed of light c, elementary charge e, Planck constant h, Avogadro constant N_A, and Boltzmann constant k.[45]

The accuracy and consistency in the determination of the values of the relevant constants, such as e, h, N_A, k, are the litmus test of whether the time for the redefinition has come.

The current adjustment is not the final one prior the redefinition. There will be an adjustment of the fundamental constants to provide the values to be used in the revision of the International System of Units expected to take place in 2018. However, the current LSA seems to be crucial for deciding whether to proceed with the redefinition or not. Once the decision is taken and the schedule is confirmed, the new adjustment is to take place. To be considered for use in this adjustment, new results must be accepted for publication by July 1, 2017.

That adjustment will be targeting the redefinition only. Still, the question of what are the best values of the constants in the "new SI units" will remain. To address this question, the 2018 CODATA adjustment of the fundamental constants, based on the revised SI, is scheduled with a standard interval of four years from the current 2014 adjustment, however with a closing date for data to be considered of July 1, 2018.

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