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Viscosity of Liquid Water in the Range $-8°C$ to 150°C

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The paper re-analyzes the results of earlier, very precise measurements of the viscosity of water at essentially atmospheric pressure. This is done in terms of a new, theoretically-based equation for the operation of a capillary viscometer rather than in terms of semi-empirical equations used by the original authors. The new analysis eliminates possible systematic errors and permits the establishment of realistic error bounds for water in its role as a standard reference substance for viscosity. The latter are smaller than those embodied in the most recent International Formulation. Standard values of the ratio of viscosity at a temperature T to its value at 20 °C have been derived from the re-analyzed data because the uncertainity of this ratio is an order of magnitude smaller than that of the absolute values. The ratios are used to generate absolute values with the aid of the standard NBS datum $\mu = 1002.0$ μPa s at 20 °C. The viscosity ratios have been correlated with the aid of two empirical equations. The more accurate equation covers the range $0\ °C \leq t \leq 40\ °C$ with an uncertainty of $\pm 0.05\%$. The less accurate equation covers the wider range $-8\ °C \leq t \leq 150\ °C$ with the more limited accuracy of $\pm 0.2\%$. The two empirical equations are compatible with each other to 0.09%.

Key words: Critically evaluated data; critical review; data compilation; liquid water; viscosity.

1. Introduction

In view of the importance of water, particularly when used as a reference substance in viscometry, the establishment of standard values for its viscosity, of well-defined accuracy over as wide a temperature range as possible, is essential. Under the guidance of the International Association for the Properties of Steam considerable progress towards this end has been made recently [1, 2].[1] In particular, a consensus of experimental evidence and opinion with regard to the viscosity of water at 20° C and a pressure of 0.1 MPa has led to the recommendation that the value μ (20 °C) = 1002.0 μPa s be adopted as a standard [1, 3, 4]. It is adjudged that this standard value has an accuracy of $\pm 0.1\%$. In the case of viscosity data at low pressures and at other temperatures the recommended correlation [1] has been based upon a number of extremely precise, recent measurements in the temperature range $0\ °C \leq t \leq 100\ °C$ [5–8].

In each case, these precise measurements were analyzed by means of a semi-empirical working equation not justified by rigorous theoretical analysis. As a consequence, it cannot be confidently asserted that the precision of the raw data is equivalent to the accuracy of the reported viscosities $\mu(t)$, or viscosity ratios $\mu(t)/\mu(20\ °C)$.

In order to derive the maximum benefit from the precision of these recent viscosity measurements the present paper presents a re-analysis of the raw experimental data by means of a rigorous working equation with a two-fold objective. First, standard values of the viscosity ratios $\mu(t)/\mu(20\ °C)$ have been obtained which are free from any systematic error introduced by the use of an incorrect working equation, and secondly, the uncertainties in these ratios have been carefully assessed with respect to the new working equation. As a result of this new treatment of the experimental data, it has been possible to generate viscosity data for liquid water over an extended temperature range ($-8\ °C \leq t \leq 150\ °C$) whose associated uncertainty is commensurate with that of the 20 °C standard value over much of the range and is well characterized elsewhere. Over the entire temperature range of the study it has been possible to reduce the uncertainty bound associated with the standard viscosity values below that embodied in the International Formulation [1, 2].

Four independent sets of data have been selected for this study on the basis that they are each demonstrably the most precise in a particular temperature range. Two determinations of the viscosity of water [5, 6] have been performed in Cannon, Rankine-type viscometers, in which the essential measurement is that of efflux time, τ, of a prescribed volume of fluid through a tube of known dimensions. In both investigations this efflux time was determined electronically and with great precision. The temperature of both instruments was also controlled very precisely, and was measured to within $\Delta t = \pm 0.001$ °C. The study

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[1] Figures in brackets indicate literature references at the end of this paper.

carried out by Korson, Drost-Hansen, and Millero [5] extended over the temperature range 10–70 °C, whereas that of Zicher and Zwolinski [6] examined the range −8 °C to 40 °C.

A second group of two papers describes measurements of lower precision but performed over an extended temperature range. The older measurements of Hardy and Cottington [7] cover the range from 5° to 125 °C, and were performed in a Bingham viscometer up to 95 °C and a Rankine viscometer above this temperature. The more recent measurements by Korosi and Fabuss [8] were performed in a Rankine viscometer over the temperature range 40° to 150 °C.

All four sets of measurements were therefore performed in variants of the capillary viscometer for which an improved, theoretically-based working equation has recently been developed [9]. Consequently, the improved theory for the viscometers provides the means for carrying out a new analysis of the data. In the case of the first group of measurements a complete reanalysis has been possible since the raw experimental data are available, thus entirely new viscosity ratios $\mu(t)/\mu(20\ ^\circ\mathrm{C})$ have been computed, together with reliable estimates of accuracy. For the second group of measurements the inaccessibility of the raw data has precluded a complete re-evaluation of the viscosities; nevertheless, the refined theory [9] has been used to obtain estimates of the accuracy of the two determinations.

The entire body of data has been employed to establish standard values of the viscosity of water at nominal temperatures and the data have been correlated by means of two empirical equations with a deviation compatible with their uncertainty.

2. The Data in the Temperature Range −8° to 70°C

2.1. Method of Re-evaluation

The re-evaluation of the data of references [5] and [6] is based upon eq (22) of ref. [9], namely

$$\tau = A\nu + B/\nu \tag{1}$$

in which A and B represent geometrical factors, that is, constants of the instruments. Explicit expressions linking these constants to the dimensions of the instrument were given in ref. [9]. By inspection, it is easy to verify that the influence of thermal expansion and buoyancy can be taken into account if A and B in equation (1) are replaced by

$$A' = A/(1-\rho_a/\rho_w)(1+\alpha\Delta t) \tag{2a}$$

and

$$B' = B(1+\alpha\Delta t)^2 \tag{2b}$$

respectively, giving

$$\tau = \frac{A\nu}{(1+\alpha\Delta t)(1-\rho_a/\rho_w)} + \frac{B(1+\alpha\Delta t)^2}{\nu}. \tag{3}$$

Here, $\nu=\mu/\rho_w$, is the kinematic viscosity of water, ρ_a is the density of air, α the linear coefficient of thermal expansion of the instrument, and Δt the difference between the temperature of the viscosity measurement and that of the determination of the instrument dimensions.

In the case of the work described in references [5] and [6] it proved impossible to establish the dimensions of the instruments with sufficient accuracy in order to calculate the constants A and B, and so to secure an absolute evaluation of results. In the circumstances, we have determined these constants with reference to the NBS data [3, 7, 10, 11] listed in table 1. The values of density were taken from references [6] and [12].

For the purposes of the evaluation of A and B, values of the viscosity at the actual temperatures found during a single measurement (which differed by at most 0.008 °C from the nearest nominal value in table 1) were generated by means of a small temperature correction applied to the values of table 1 according to the equation [10]

$$\log\left\{\frac{\mu(t)}{\mu(t^*)}\right\} = \frac{20-t}{t+96}\left[1.2348-0.001467(20-t)\right] - \frac{20-t^*}{t^*+96}\left[1.2348-0.001467(20-t^*)\right]. \tag{4}$$

We do not thereby imply that eq (4) necessarily represents the variation of the viscosity of water with temperature with utmost precision, but merely that it is sufficiently accurate to allow the small correction to the viscosity between the measurement at t and the value at the nominal temperature t^*.

2.2. Re-evaluation of the Data of L. Korson, W. Drost-Hansen, and F. J. Millero [5]

The authors of reference [5] evaluated their experimental results with the aid of a theoretically inspired, but essentially empirical, correlation formula

$$\nu = \tau K(1-\rho_a/\rho_w) - L/\tau^n, \tag{5}$$

in which the adjustable constants K, L, and n were determined with reference to the values of the viscosity of water at 20 °C, 25 °C, 30 °C, 40 °C, 50 °C, and 60 °C generated at the National Bureau of Standards [3, 7, 10, 11]. Thus table 2 in reference [5] constitutes a largely empirical interpolation and extrapolation of the viscosity of water in the range 0 °C–100 °C.

TABLE 1. Values of the viscosity of water recommended by NBS [3, 7, 10, 11]

Temperature t °C	20	25	40	60
Viscosity μ, μPa s	1002. 0	890. 3	652. 6	466. 6

With the aid of the original data supplied by Professor W. Drost-Hansen, and the reference values of viscosity contained in table 1, we have determined the constants A and B of equation (3) for his apparatus by a least-squares analysis according to the relation,

$$\nu\tau = A'\nu^2 + B',$$

with A' and B' defined by equations (2a) and (2b).

This analysis yielded the results

$$\left.\begin{aligned} A &= (2.27234 \pm 0.00071) \times 10^8\,\mathrm{s^2/m^2} \\ B &= (6.7 \pm 0.53) \times 10^{-7}\,\mathrm{m^2}, \end{aligned}\right\} \qquad (6)$$

where the quoted accuracy is measured in terms of the respective standard deviation. In this analysis the data at 25 °C did not seem to conform to the required linear relationship as perfectly as the other points and for this reason, and suspecting an undetected experimental difficulty, we have omitted them from the statistical fit.

Having determined the optimum values of A and B for eq (3) we recomputed all the original data. These were once again corrected to nominal temperatures with reference to eq (4) and averaged at each nominal temperature. The final result is contained in table 2 which lists the optimum values of the absolute viscosity, μ, the density from refs. [6] and [12] and the corresponding value of the kinematic viscosity. In each case the uncertainties take into account the maximum random error, $\Delta\tau = 0.003$ s, in the measurement of time, as reported in ref. [5]. In addition, the effect of uncertainties in the determination of the constants A and B from eq (3), as given in eq (6), are included. In this manner, our estimate of uncertainty makes an allowance for the imperfections in the reproducibility of measurements in the instrument. The latter, reflected in the original data, is presumed to be caused by unaccounted for random errors in the triggering of the electronic timer. The density has been assumed to be known without error, and the small buoyancy correction $(1-\rho_a/\rho_w)$ has also been disregarded as far as the error estimate is concerned.

A study of table 2 reveals that the overall accuracy of the dynamic and kinematic viscosity varies from ±0.1% at 10 °C to 2.4% at 70 °C, assuming that the NBS values from table 1 are exact. The main source of this error stems from the imperfect reproducibility of the instrument. It is noteworthy that our re-evaluated data do not differ from those reported in table 2 of reference [5] by more than 0.2%. Hence, within the uncertainty given, the two tabulations must be considered equivalent, thus justifying the use of the semi-empirical equation (5) in this context.

Whereas the determination of the actual value of the kinematic viscosity, $\nu(t)$, is accurate to within a few parts in one hundred, it is still possible to determine the ratios $\nu(t)/\nu(20\,°\mathrm{C})$ with a reduced uncertainty. Such calculations can be based on the equation [9]:

$$\left.\begin{aligned} \frac{\nu(t)}{\nu(20\,°\mathrm{C})} &= \frac{\tau + [\tau^2 - 4ABg(t)]^{1/2}}{\tau_{20\,°\mathrm{C}} + [\tau^2_{20\,°\mathrm{C}} - 4ABg(20\,°\mathrm{C})]^{1/2}}, \\ g(t) &= \frac{1+\alpha\Delta t}{[1-\rho_a(t)/\rho_w(t)]}, \end{aligned}\right\} \qquad (7)$$

as required by equation (2) to account for the buoyancy and thermal expansion effects. Equation (7) now contains a single instrument constant, $4AB$. The latter has been determined with reference to the NBS measurements at 40 °C and 60 °C and an average taken. The process yielded,

$$4AB = (699 \pm 154)\,\mathrm{s^2}. \qquad (8)$$

Because the typical flow times for this investigation were of the order of 200 s, it is clear that the deviations for the simplified formula

$$\frac{\nu(t)}{\nu(20\,°\mathrm{C})} = \frac{\tau}{\tau_{20\,°\mathrm{C}}} \qquad (9)$$

were very slight in the measurements. With the aid of the value of $4AB$ given in eq (8) we have re-evaluated all the viscosity ratios for the work of reference [5]. The results are given in table 3 together with the uncertainties in the values. The errors in the ratios are now of the order of a few parts per thousand at worst and are therefore much less than the uncertainty in the values of the viscosity taken separately.

2.3. The Re-evaluation of the Data of L. D. Eicher and B. J. Zwolinski [6]

The measurements in reference [6] were evaluated originally with the aid of the standard formula

$$\nu = C(1+\alpha\Delta t)M\tau - \frac{E(1+\alpha\Delta t)^{5/2}}{\tau^2}, \qquad (10)$$

where C and E are instrument constants and M is, essentially, a buoyancy correction. This equation also differs from the more accurate relation (3). Employing the latter together with the values of viscosity at 20° C, 25 °C and 40 °C listed in table 1 we obtain, following our earlier procedure,

$$\begin{aligned} A &= (3.0813 \pm 0.0024) \times 10^8\,\mathrm{s^2/m^2} \\ B &= (4.55 \pm 0.23) \times 10^{-7}\,\mathrm{m^2}, \end{aligned} \qquad (11)$$

whereas the use of eq (7) for the viscosity ratio leads to

$$4AB = (437 \pm 137)\,\mathrm{s^2}. \qquad (12)$$

Following the pattern adopted in the previous section we have again recalculated the original data and obtained the results for absolute viscosities given in table 4, and the viscosity ratios of table 5. In each case the estimated uncertainty is included in the

TABLE 2. Re-evaluation of viscosities from the original data of L. Korson, W. Drost-Hansen, and F. J. Millero [5]

Temp. °C	Density ρ, kg/m^3	Viscosity μ, μPa s from [5]	Viscosity μ, μPa s	Kinematic viscosity ν, mm^2/s	Uncertainty $\Delta\nu$	Relative uncertainty $\Delta\mu/\mu$ or $\Delta\nu/\nu$, %	Deviation from [5] %
10	999.7281	1306.9	1307	1.307	0.008	0.06	−0.02
15	999.1286	1138.2	1138	1.139	0.0015	0.13	−0.03
20	998.2336	1002.0	1002	1.003	0.0022	0.22	−0.01
25	997.0751	890.3	889	0.892	0.0031	0.35	−0.10
30	995.6783	797.5	796	0.800	0.0039	0.49	+0.18
35	994.0635	719.5	718	0.723	0.0047	0.65	+0.15
40	992.2473	653.5	652	0.658	0.0058	0.89	+0.15
45	990.24	596.3	595	0.601	0.0064	1.08	+0.16
50	988.07	547.1	547	0.553	0.0071	1.30	+0.11
55	985.73	504.2	504	0.511	0.0078	1.55	+0.03
60	983.24	466.6	467	0.475	0.0085	1.82	−0.09
65	980.59	433.4	434	0.443	0.0092	2.12	−0.17
70	977.81	403.9	405	0.414	0.0098	2.42	−0.26

$A=(2.2734\pm0.00071)\times10^8\ \mathrm{s^2/m^2}$.
$B=(6.7\pm0.53)\times10^{-7}\mathrm{m}^2$.

TABLE 3. Re-evaluation of original data of L. Korson, H. Drost-Hansen, and F. J. Millero [5] on a relative basis

Reference temperature: 20 °C

Temp. t °C	Density ratio $\rho(t)/\rho(20\ ^\circ\mathrm{C})$ [12]	Viscosity ratio $\mu(t)/\mu(20\ ^\circ\mathrm{C})$	Ratio of kinematic viscosities $\nu(t)/\nu(20\ ^\circ\mathrm{C})$	Uncertainty $\Delta[\nu(t)/\nu(20\ ^\circ\mathrm{C})]$	Relative uncertainty %
10	1.00150	1.3403	1.3024	0.0003	0.02
15	1.00090	1.1361	1.1351	0.0001	0.01
20	1.00000	1.00000	1.00000	0.00000	0.00
25	0.99884	0.8876	0.8886	0.0001	0.02
30	0.99744	0.7945	0.7966	0.0002	0.03
35	0.99582	0.7171	0.7201	0.0004	0.05
40	0.99400	0.6510	0.6549	0.0005	0.07
45	0.99199	0.5943	0.5991	0.0006	0.10
50	0.98982	0.5456	0.5512	0.0007	0.13
55	0.98747	0.5033	0.5097	0.0008	0.15
60	0.98498	0.4664	0.4736	0.0009	0.19
65	0.98233	0.4336	0.4414	0.0010	0.22
70	0.97954	0.4045	0.4130	0.0011	0.27

$4AB=(669\pm154)\ \mathrm{s}^2$.

tabulation. The uncertainty for the absolute viscosity does not exceed ±0.1% assuming no error in the NBS values, whereas that in the viscosity ratio is less than 0.05%. The values included in tables 4 and 5 depart by less than 0.06% from those listed in reference [6] justifying the use of the approximate equation (10) for analysis.

3. The Data in the Extended Temperature Range

Although both Korosi and Fabuss [8] and Hardy and Cottington [7] used Rankine-type viscometers for their measurements, their published work does not contain sufficient information to permit a re-evaluation of their data with the aid of the more accurate working equation. Nevertheless, it is possible to estimate values of both A and B from their work which are sufficient to allow a statement regarding the likely effect of the analysis of their results with an empirical working equation.

For the work of Korosi and Fabuss [8] the characteristics of the two viscometers used indicate that the values of the instrument constants are approximately

$$\left.\begin{aligned}A&=4\times10^8\mathrm{s^2/m^2}\\B&=3\times10^{-7}\mathrm{m^2}\end{aligned}\right\}\tag{13}$$

for the low temperature measurements, and

$$\left.\begin{aligned}A&=1.7\times10^9\mathrm{s^2/m^2}\\B&=4\times10^{-7}\mathrm{m^2}\end{aligned}\right\}\tag{14}$$

for the high temperature viscometer.

TABLE 4. Re-evaluation of viscosities from the original data of L. D. Eicher and B. J. Zwolinski [6] with the aid of eq (3)

Temp. °C	Density ρ, kg/m^3	Viscosity μ, μPa s from [6]	Re-evaluation				
			Viscosity μ, μPa s	Kinematic viscosity ν, mm^2/s	Uncertainty $\Delta\nu$	Relative uncertainty $\Delta\mu/\mu$ or $\Delta\nu/\nu$, %	Deviation from [6] %
−8.280	0.998502	2455.1	2456.6	2.4603	0.0002	0.01	−0.06
−6.647	0.998878	2296.0	2297.3	2.2999	0.0002	0.01	−0.06
−4.534	0.999329	2113.7	2114.9	2.1164	0.0002	0.01	−0.06
−1.108	0.999752	1863.0	1863.9	1.8644	0.0002	0.01	−0.05
0	0.9998396	1791.9	1792.7	1.7930	0.0002	0.01	−0.04
5	0.9999641	1519.0	1519.6	1.5196	0.0002	0.01	−0.04
10	0.9997281	1306.7	1307.1	1.3075	0.0002	0.01	−0.03
15	0.9991286	1138.0	1138.4	1.1393	0.0002	0.01	−0.04
20	0.9982336	1002.0	1002.1	1.0039	0.0001	0.02	−0.01
25	0.9970751	890.20	890.2	0.8928	0.0001	0.02	0.0
30	0.9956783	797.23	797.2	0.8006	0.0002	0.02	0.01
35	0.9940635	719.11	719.0	0.7233	0.0002	0.02	0.01
40	0.9922473	652.70	652.7	0.6578	0.0002	0.03	0.01

$A=(3.0813\pm0.00024)\times10^8$ s^2/m^2.
$B=(4.55\pm0.23)\times10^{-7}$ m^2.

TABLE 5. Re-evaluation of original data of L. D. Eicher and B. J. Zwolinski [6] on a relative basis
Reference temperature: 20 °C

Temp. t °C	Density ratio $\rho(t)/\rho(20\ ^\circ\mathrm{C})$	Viscosity ratio $\mu(t)/\mu(20\ ^\circ\mathrm{C})$	Ratio of kinematic viscosities $\nu(t)/\nu(20\ ^\circ\mathrm{C})$	Uncertainty $\Delta[\nu(t)/\nu(20\ ^\circ\mathrm{C})]$	Relative uncertainty %
−8.280	1.00027	2.4508	2.4502	0.0007	0.03
−6.647	1.00065	2.2920	2.2905	0.0007	0.03
−4.534	1.00110	2.1103	2.1080	0.0006	0.03
−1.108	1.00152	1.8596	1.8568	0.0005	0.03
0	1.00161	1.7886	1.7857	0.0004	0.03
5	1.00173	1.5161	1.5135	0.0003	0.02
10	1.00150	1.3042	1.3023	0.0002	0.02
15	1.00090	1.1359	1.1348	0.0001	0.01
20	1.00000	1.0000	1.0000	0.0000	0.00
25	0.99884	0.8884	0.8894	0.0001	0.01
30	0.99744	0.7957	0.7977	0.0002	0.02
35	0.99582	0.7178	0.7208	0.0002	0.03
40	0.99400	0.6516	0.6555	0.0003	0.05

$4AB=(437\pm134)$ s^2.

For both instruments, an approximate calculation indicates that the absolute viscosities reported could have a maximum error arising from the use of an inaccurate working equation of the order of 0.1%. On the other hand, the errors introduced into the viscosity ratios $\nu(t)/\nu(20\ ^\circ\mathrm{C})$ would be almost an order of magnitude smaller. Thus, provided we use the reported viscosity ratios [8] there is no need to adjust the uncertainty band reported by Korosi and Fabuss [8] as one of ±0.2%.

For the instrument employed by Hardy and Cottington [7] the approximate values of A and B are

$$A=8.1\times10^8\ \mathrm{s^2/m^2}$$

and

$$B=1.2\times10^{-6}\ \mathrm{m^2}.$$

These values are somewhat larger than those characteristic of the other instruments. However, the authors have not provided sufficient information about their own working equation

$$\nu=A''\tau-B''/\tau \qquad (14a)$$

to allow even an assessment of the uncertainty introduced because of its use. Consequently we can only infer that the uncertainties introduced in their reported absolute viscosities are probably greater than ±0.3% at the highest temperatures, and that their relative viscosities may be burdened with an additional uncertainty of as much as ±0.27%.[2]

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[2] Hardy and Cottington [7] employed a reference value for viscosity of 1005.0 μPa s at 20 °C. The viscosity ratios reported here are derived directly as $\mu(t)/\mu(20\ ^\circ\mathrm{C})$ from their data, since insufficient information is available to allow adoption of any other action.

The viscosity ratios $\mu(t)/\mu(20\ ^\circ\mathrm{C})$ for these two sets of measurements are given in table 6. On the basis of the foregoing discussion and the estimated uncertainties reported by the authors, accounting for the random errors of measurement, both sets of data are estimated to have uncertainty of $\pm 0.3\%$.

4. The Viscosity Ratio $\mu(t)/\mu(20\ ^\circ\mathrm{C})$ for Water

Sections 2 and 3 have demonstrated that the capillary viscometer is capable of providing values of the ratio of viscosities $\mu(t)/\mu(t')$ much more accurately than values of the absolute viscosities. In addition, because the value of the viscosity at 20 °C, 1002.0 μPa s, is the most accurately known, the proper use of the foregoing analysis is first to establish a set of standard viscosity ratios and then to generate absolute viscosities.

Accordingly we have employed the viscosity ratios in tables 3, 5, and 6 for this purpose. They have first been divided into two temperature ranges. In the first range $0\ ^\circ\mathrm{C} \leq t \leq 40\ ^\circ\mathrm{C}$ the recalculated data of references [5] and [6] have an estimated uncertainty which does not exceed 0.05%. These values can be correlated by means of the empirical equation

$$\log\left\{\frac{\mu(t)}{\mu(20\ ^\circ\mathrm{C})}\right\} = \frac{20-t}{t+96}\{1.2364 - 1.37\times 10^{-3}(20-t) + 5.7\times 10^{-6}(20-t)^2\}\quad 0\ ^\circ\mathrm{C} \leq t \leq 40\ ^\circ\mathrm{C}, \qquad (15)$$

where t is measured in degrees Celsius. With the exception of three points due to Korson et al. [5], one of which has been discussed earlier, equation (15) describes all the data within their uncertainty as shown in the deviation plot of figure 1; the standard deviation is 0.02%.

TABLE 6. Viscosity ratios for the data of R. C. Hardy and R. L. Cottington [7] and A. Kosrosi and B. M. Fabuss [8]

Reference temperature: 20 ° C

Temperature t °C	Viscosity ratio [7] $\mu(t)/\mu(20\ ^\circ\mathrm{C})$	Viscosity ratio [8] $\mu(t)/\mu(20\ ^\circ\mathrm{C})$
20	1. 0000	1. 0000
40	0. 6518	0. 6514
60	0. 4656	0. 4656
75		0. 3776
80	0. 3577	
90	0. 3142	
95	0. 2970	
100	0. 2816	0. 2814
110	0. 2544	
120	0. 2316	
125	0. 2216	0. 2215
135		0. 2036
145		0. 1880
150		0. 1811

Estimated uncertainty $\Delta[\mu(t)/\mu(20\ ^\circ\mathrm{C})] = \pm 0.3\%$.

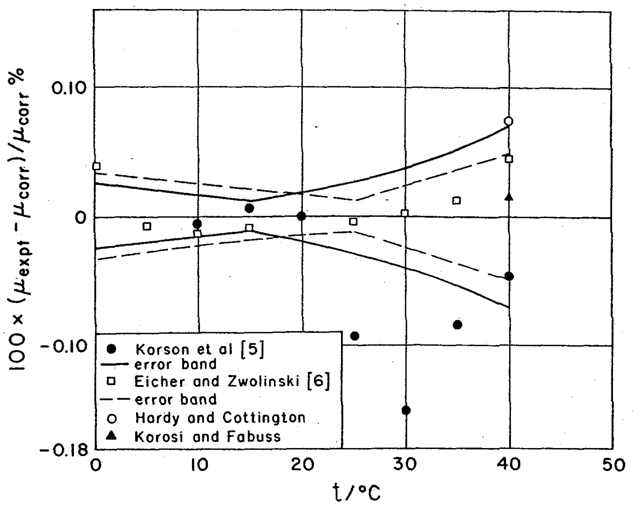

FIGURE 1. Deviations of the re-calculated experimental data of references [5], [6], [7] and [8] from the correlating equation (15). Standard deviation: 0.05%.

Over the extended temperature range $-8\ ^\circ\mathrm{C} \leq t \leq 150\ ^\circ\mathrm{C}$ the entire body of data has been correlated by the equation

$$\log\left\{\frac{\mu(t)}{\mu(20\ ^\circ\mathrm{C})}\right\} = \frac{20-t}{t+96}\{1.2378 - 1.303\times 10^{-3}(20-t) + 3.06\times 10^{-6}(20-t)^2 + 2.55\times 10^{-8}(20-t)^3\}. \qquad (16)$$

The deviations of the tabulated data from this correlation are shown in figure 2. The maximum deviation is one of $\pm 0.26\%$, and is therefore commensurate with the error band of the data included in this complete correlation.

5. Tabulation

Equations (15) and (16) have been used to generate values for the absolute viscosity of water over the temperature range −8 °C to 150 °C. Over the tem-

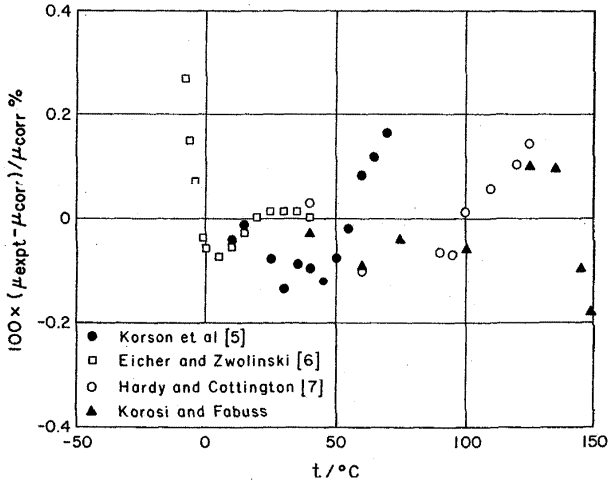

FIGURE 2. Deviations of re-calculated experimental data of references [5], [6], [7] and [8] from the correlation equation (16). Standard deviation: 0.09%.

perature range 0 °C to 40 °C equation (15) has been used together with the value $\mu(20\ ^\circ\mathrm{C}) = 1002.0\ \mu\mathrm{Pa\ s}$. If the uncertainty in this reference value is taken as ±0.1% then the uncertainty of the tabulated value is ±0.15% for this range. Outside this limited temperature range, equation (16) has been used together with the same reference viscosity, the uncertainty in the tabulated value is here ±0.4%.

These tabulated viscosities have been compared with those calculated from the Setpember 1975, Release on the Dynamic Viscosity of Water Substance, issued by the International Association for the Properties of Steam [2], and figure 3 contains a plot of the deviations between the two sets of values. Over the temperature range $0\ ^\circ\mathrm{C} \leq t \leq 40\ ^\circ\mathrm{C}$ the deviations do not exceed ±0.1%, whereas over the temperature range to 150 °C the departures amount to ±0.3%. A comparison of the present correlations with that given by Nagashima [1] reveals that the discrepancies never exceed ±0.5% in the temperature range 0° to 100 °C. This agreement is in all cases within the mutual uncertainty of the correlations, however, the present work has allowed the viscosity values to be quoted with closer tolerances than possible hitherto.

As a matter of additional interest, we provide a comparison of the two correlations contained in equations (15) and (16) in the overlapping region $0\ ^\circ\mathrm{C} \leq t \leq 40\ ^\circ\mathrm{C}$, stressing that the former is considerably more reliable in this temperature range. The comparison has been made graphically in figure 4 in the form of a plot of the percentage deviation of $\mu_{(16)} - \mu_{(15)}$ from $\mu_{(16)}$. Here, the subscript refers to the equation used for the calculation of the respective quantity. It is seen that the maximum deviation of 0.09% is well within the 0.26% deviation characteristic of eq (16).

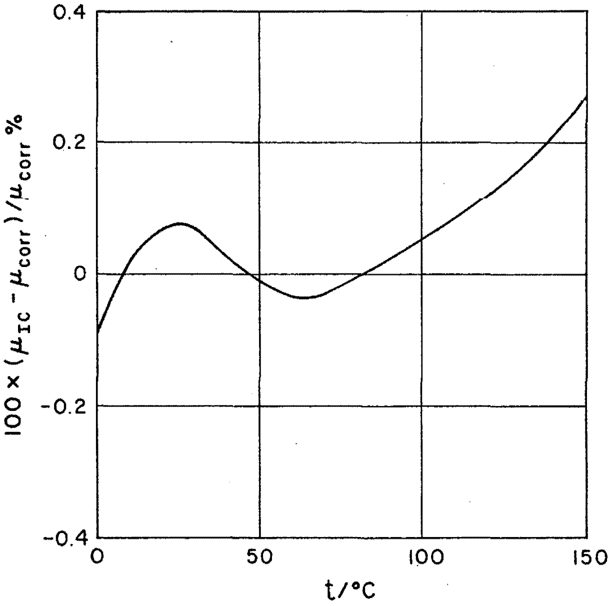

FIGURE 3. Deviations of the International Formulation for the viscosity of water, μ_{IC}, from the present correlating equation (16).

TABLE 7. Standard viscosities of liquid water from equation (15) and (16)

Temperature t °C	Viscosity μ(μPa s)	Temperature t °C	Viscosity μ(μPa s)
−8	2421.0	75	378.5
−5	2144.0	80	355.1
0	1791.5	85	334.1
5	1519.3	90	315.0
10	1307.0	95	297.8
15	1138.3	100	282.1
20	1002.0	105	267.8
25	890.2	110	254.4
30	797.3	115	242.3
35	719.1	120	231.3
40	652.7	125	221.3
45	596.1	130	212.0
50	547.1	135	203.4
55	504.4	140	195.5
60	467.0	145	188.2
65	433.9	150	181.4
70	404.6		

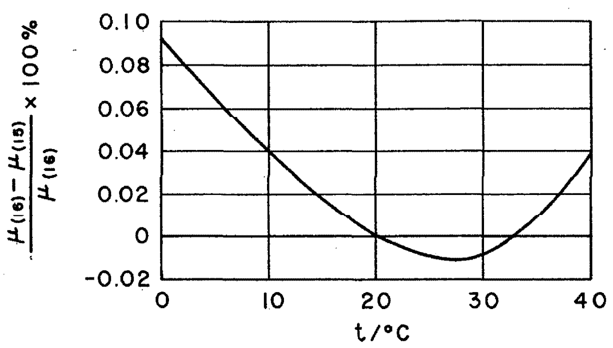

FIGURE 4. Deviation between the values given by the two correlating equations (15 and (16). Maximum deviation: 0.09%.

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