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Static Dielectric Constant of Water and Steam

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This paper reviews and evaluates the experimental works of the static dielectric constant (permittivity) of water and steam over the century. The critically evaluated experimental data are represented by a function of temperature and density. This representation was carefully examined in the light of the criteria for smoothness and physical plausibility. As a result of this work, which was largely stimulated by the activities of the International Association for the Properties of Steam, a new International formulation for the static dielectric constant of water and steam was adopted. This formulation covers a temperature range from 0 to 550 °C and a pressure range up to 500 MPa.

Key words: Critically evaluated data; critical review; data compilation; International Formulation; static dielectric constant; steam; water.

1. Introduction

As a part of the activities of the International Association for the Properties of Steam (IAPS) to formulate and standardize data on the thermophysical properties of water and steam for industrial and scientific applications, an international agreement has been reached concerning the representation of the static dielectric constant (permittivity), which is valid for electric fields of very low frequency and moderate intensity, of water substance in the range of temperatures 0 to 550 °C and of pressures up to 500 MPa. This representation was prepared by Working Group III (WG-III) of IAPS[1] as designated by the 8th International Conference on the Properties of Steam (ICPS) held in France in 1974. The WG-III met in Schliersee, Germany, in April 1975, in Ottawa, Canada, in September 1975, in Kyoto, Japan, in September 1976, and in Moscow, USSR, in September 1977, and adopted a final recommendation concerning the static dielectric constant. The actual task of reviewing the experimental data and formulating a representative equation was conducted by the present authors, when one of them (MU) worked in the Institute of Physical Chemistry of the University of Karlsruhe.

This final recommendation was duly circulated among the national delegations represented at the 8th ICPS and approved in 1978 by mail ballot. It is now embodied in an official document entitled Release on Static Dielectric Constant of Water Substance which was circulated among a large number of technical journals of the world. Thus, this recommendation has officially acquired the status of an international standard.

The Release is given verbatim in its officially approved form as an Appendix in this paper. The main part of the paper describes the present state of our knowledge concerning the static dielectric constant of water substance and gives details concerning both evaluation of the experimental data and correlation of these data.

2. Available Data Sources

The static dielectric constant (ϵ) for liquid water under atmospheric pressure has been measured repeatedly since the end of last century. Two points of experimental ϵ-values at 16 °C up to 20 MPa appeared in 1920 [1][2] as the earliest data at high pressures. Since then, as shown in table 1 the range of experiments was extended to 600 °C and 2 GPa, respectively. Dorsey compiled an almost complete survey [2] of the literature published till 1940.

The physical theory of the dielectric behavior of polar fluids and in particular the dielectric constant of water has been treated thoroughly and extensively [3–6]. Based on the earlier work of Debye, Onsager, Fröhlich, and Kirkwood [7], more recent theoretical advances are due to Buckingham et al. [8], Wertheim [9], Deutch [10] and Stillinger et al. [11]. Methods and equations to estimate the dielectric constant of water over a wide range of temperatures and pressures were proposed by Franck [12], Quist and Marshall [13], and Jansoone and Franck [14].

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[1] The WG-III consisted of representatives from the Federal Republic of Germany, France, Japan, UK, the USA, and the USSR.

[2] Figures in brackets indicate literature references.

Table 1. Experimental data sources

First Author	Year	Experimental Range temp. (°C)	pres. (MPa)	Claimed Accuracy (%)	Presentation	Reference
Cohn	(1892)	9 -- 35	0.1		D	[15]
Heerwagen	(1893)	5 -- 21	0.1		D	[16]
Ratz	(1896)	4 -- 30	0.1		D	[17]
Coolidge	(1899)	4 -- 39	0.1		D	[18]
Falckenberg	(1920)	16	0.7 -- 20		D	[1]
Powers	(1920)	22	0.1		D	[19]
Sauzin	(1920)	17	0.1		D	[20]
Jezewski	(1920)	15	0.1		D	[21]
Lattey	(1921)	18	0.1		D	[22]
Jezewski	(1922)	2 -- 100	0.1		D	[23]
Carman	(1924)	25	0.1		D	[24]
Kockel	(1925)	0 -- 100	0.1	0.8	D	[25]
Matsuike	(1925)	25	0.1	0.1	D	[26]
Kyropoulos	(1926)	20	0.1 -- 300	2	D	[27]
Deubner	(1927)	17	0.1		D	[28]
Astin	(1929)	25	0.1		D	[29]
Cuthbertson	(1930)	0 -- 75	0.1	1	D	[30]
Drake	(1930)	3 -- 60	0.1		E	[31]
Devoto	(1930)	4 -- 40	0.1		D	[32]
Wyman	(1930)	0 -- 100	0.1	0.2	E	[33]
Linton	(1931)	25	0.1	1	D	[34]
Lattey	(1931)	10 -- 80	0.1		D	[35]
Åkerlöf	(1932)	10 -- 100	0.1		D	[36]
Linton	(1932)	0 -- 50	0.1	0.2	E	[37]
Albright	(1937)	18 -- 50	0.1		D	[38]
Wyman	(1938)	0 -- 100	0.1		E	[39]
Jones	(1939)	20 -- 25	0.1	0.3	D	[40]
Albright	(1946)	5 -- 55	0.1	0.1	D	[41]
Lees	(1949)	0 -- 50	0.1 -- 2000	0.01	D	[42]
Akerlof	(1950)	100 -- 364	sat. water		E	[43]
Harris	(1953)	14 -- 75	0.1 -- 19		D	[44]
Harris	(1953)	25	0.1 -- 13		D	[45]
Fogo	(1954)	378 -- 397	23 -- 28	2	D	[46]
Scaife	(1955)	20	0.1 -- 600	4	D	[47]
Malmberg	(1956)	0 -- 100	0.1	0.1	D	[48]
Owen	(1961)	0 -- 70	0.1 -- 100		E	[49]
Vidulich	(1962)	0 -- 40	0.1	0.05	D	[50]
Gier	(1963)	200 -- 350	1.5 -- 200		D	[51]
Rusche	(1966)	-5 -- 25	0.1		E	[52]
Vidulich	(1967)	0 -- 40	0.1		D	[53]
Koslov	(1967)	310 -- 460	21 -- 35	20	G	[54]
Dunn	(1969)	5 -- 65	0.1 -- 200		D	[55]
Kay	(1969)	25	0.1		D	[56]
Schadow	(1969)	20 -- 45	0.1 -- 130	0.05	D	[57]
Heger	(1969)	100 -- 550	0.1 -- 500	0.3	D	[58]
Svistunov	(1974)	250 -- 370	sat. steam		G	[59]
Golubev	(1974)	150 -- 370	sat. steam		G	[60]
Bychkov	(1974)	-45 -- 25	0.1		G	[61]
Srinivasan	(1974)	10 -- 40	0.1 -- 300	0.03	E	[62]
Lukashov	(1975)	400 -- 600	18 -- 60		D	[63]

Presentation:

D: numerically, E: by an interpolation equation, G: graphically

2.1. Static Dielectric Constant of Water at Atmospheric Pressure in the Range 0–100 °C

Eighty-four percent of the 50 measurements listed in table 1 were made at atmospheric pressure and those published till 1930 were measured at room temperatures with great uncertainty. The data in [33,36,38,41,48,49,50,52,53,55,56,62] coincide well with each other within a difference of $\pm 0.5\%$. The measurements of Wyman [33] covering the range of temperatures from 0 to 100 °C were made by the resonance method with an accuracy of 0.2%, those of Åkerlöf [36] from 10 to 100 °C by the resonance method, those of Albright et al. [38,41] from 5 to 55 °C by the bridge method with an accuracy of 0.1%, those of Malmberg and Maryott [48] from 0 to 100 °C by the bridge method with an accuracy of 0.1%, those of Owen et al. [49] from 0 to 70 °C by the resonance method, those of Kay et al. [50,53,56,62] from 0 to 40 °C by the bridge method with an accuracy of 0.05%, those of Rusche and Good [52] from –5 to 25 °C by the bridge method and those of Dunn and Stokes [55] from 5 to 65 °C by the bridge method.

Wyman and Ingalls [39] published the other set of data in 1938 and these data are systematically larger by about 0.3% than those by Malmberg and Maryott. The difference between the data in [39] and in [33] increases with temperature from 40 °C and reaches a maximum at 100 °C where the value of [39] is larger by about 1% than that of [33]. Deviations of the data by Kockel [25], by Devoto [32] and by Jones and Davies [40] from other data decrease with increasing temperature, whereas those of Drake et al. [31] increase. The data by Jezewski [23], by Maass et al. [30,34,37], and Lattey [35] scatter greatly in comparison with the others. The data in [61] are given in terms of graphical presentation and are, therefore, inconvenient as material for a precise analysis.

2.2. Static Dielectric Constant of Water and Steam under High Pressures

Experimental data for ϵ of water and steam in the high pressure region have been reported in 18 papers [1,27,42–47,49,51,54,55,57–60,62,63]. A plot of ϵ vs temperature was made to aid in preliminary screening of the data visually. The isotherms given by Gier and Young [51] were completely different from others as pointed out previously in literature [13,58]. Then the available experimental data were compared at various conditions of temperature and pressure. The data of Kyropoulos [27], of Scaife [47] and of Schadow and Steiner [57] deviated widely from those of the remaining measurements. The temperature dependence of the data by Harris et al. [44,45] was slightly different from that of the others. The earliest investigation [1] gave only 2 points of ϵ–data in liquid water at 16 °C, where many measurements have been reported up to now. The graphical presentation for ϵ–data, which is not convenient as material for a precise analysis, was given by three papers [54,59,60]. The rest of data sources [42,43,46,49,55,58,62,63] cover almost the whole range of temperatures from 0 to 550 °C and of pressures up to 500 MPa as shown in figure 1.

The measurements of Lees [42] covering the range between 0 and 50 °C up to 2 GPa were made by the bridge method with an accuracy of 0.01%, and those of Akerlof and Oshry [43] between 100 and 364 °C along the saturation line of liquid water by the resonance method. Fogo et al. [46] covering the critical region of temperatures from 378 to 397 °C and of pressures from 23 to 27.5 MPa measured ϵ–values as a function of temperature and density by the bridge method with an uncertainty of 2%. The measurements of Owen et al. [49] between 0 and 70 °C up to 100 MPa were made by the resonance method, those of Dunn and Stokes [55] between 5 and 65 °C up to 200 MPa by the bridge method, those of Heger [58] between 100 and 550 °C up to 500 MPa by the bridge method with an accuracy of 0.3%, and those of Srinivasan and Kay [62] between 10 and 40 °C up to 300 MPa by the bridge method with an accuracy of 0.03%. Lukashov et al. [63] measured ϵ of steam in the range of temperatures from 400 to 600 °C and of pressures from 18 to 60 MPa by the bridge method.

3. New Formulation

3.1. Compilation and Evaluation of Data

The available experimental data from the literature were compiled and surveyed in the Institute of Physical Chemistry, University of Karlsruhe. These were presented to the meeting of WG-III at Schliersee in Germany in April 1975 [64]. These data were selected and weighting factors were assigned to them which are listed in table 2. The results were presented to the meeting of WG-III at Kyoto in Japan in September 1976 [65], and discussed with the members of WG-III.

The experimental data sources listed in table 1 were selected on the basis of the following conditions:

(a) reliability of the measuring techniques and equipment
(b) the author's own error estimate
(c) relative discrepancy among the data
(e) adequacy of the presentation of the data

The weighting factor was assigned according to the evaluated precision of the data; they were ranged on the basis of the following scale: 5, 3, 1, 0. The data of water at atmospheric pressure are more accurate than those at high pressures and a weighting factor of 10 was assigned to the data at atmospheric pressure.

3.2. Interpolation Equation and Tables

Based on the selected experimental data considering weighting factors listed in table 2, a new interpolation equation, which the Eighth ICPS adopted, was proposed by the present authors [65,66]. The equation given below describes the dielectric constant as a function of temperature and density.

$$\begin{aligned}\epsilon = 1 \quad & + (A_1 / T^*)\rho^* \\ & + (A_2 / T^* + A_3 + A_4 T^*)\rho^{*2} \\ & + (A_5 / T^* + A_6 T^* + A_7 T^{*2})\rho^{*3} \\ & + (A_8 / T^{*2} + A_9 / T^* + A_{10})\rho^{*4}\end{aligned} \tag{1}$$

where $\rho^* = \rho / \rho_0$

$T^* = T / T_0$

ρ, density in kg/m^3

T, temperature in K

A_i, ρ_0, T_0, numerical constants given in table 3

All of the investigators listed in table 2 except Fogo et al. [46] and Lukashov et al. [63] measured the ϵ-values at certain conditions of pressure and temperature. For these data, density values below 100 MPa were calculated from the 1968 IFC Formulation for Scientific and General Use [67] and those above 100 MPa were obtained from an equation of state proposed by Jůza [68]. The numerical values of the coefficients in eq (1), determined with a least-squares procedure, are fixed as given in table 3. The equation reproduces the 892 input data points with a standard deviation of 0.33 in ϵ-units. Tables 4 and 5 contain smooth values calculated with the aid of the equation. Figures 2 and 3 depict the dependence of static dielectric constant on pressure and temperature. Curves of constant ϵ are shown in figure 4 on a temperature-density diagram.

3.3. Comparison of the New ICPS Representation with Experimental Data

The new interpolation equation and tables were deduced directly from the experimental data. The deviation plots in figures 5, 6, and 7 compare the calculated values with the experimental values.

Figures 5 and 6 show comparisons with experimental data under high pressures. The values calculated by eq (1) agree with 94% of the experimental data [42,43,46,49,55,58,62,63] within a deviation of 0.4 in ϵ-units. Lees' data [42] covering the range between 0 and 50 °C up to a maximum pressure of 2 GPa deviate within 0.2 in ϵ-units corresponding to 0.2% up to 200 MPa, but deviations occur at pressures above 200 MPa. This difference in pressure dependence above 200 MPa is attributed to the revision of the pressure scale. Owen et al. [49] measured ϵ-data in the range of temperatures from 0 to 70 °C up to 100 MPa and their data deviate within 0.29 in ϵ-units (0.32%). Dunn and Stokes [55] measured ϵ-data in the range of temperatures from 5 to 65 °C up to 200 MPa and their data deviate within 0.43 in ϵ-units (0.49%). The data of Srinivasan and Kay [62] covering the range between 10 and 40 °C up to 300 MPa deviate within 0.40 in ϵ-units (0.49%) except 1 point at 25 °C and 300 MPa. The data of Lees, of Owen et al., of Dunn and Stokes, and of Srinivasan and Kay are in good agreement within 0.3% in the range of temperatures between 0 and 70 °C up to 300 MPa.

The data of Akerlof and Oshry [43] between 100 and 364 °C on the coexistence line of liquid water deviate within 0.4 in ϵ-units (1.4%) from 100 and 260 °C and within 0.7 in ϵ-units (4.1%) up to 360 °C. These data are smaller than Heger's data [58] by 2% above 300 °C, whereas larger by 0.5% below 250 °C. Heger's data covering the range from 100 to 550 °C up to 500 MPa deviate within 0.4 in ϵ-units except 3 points (−0.44, 250 °C, 5 MPa; 0.45, 400 °C, 100 MPa; 0.49, 400 °C, 150 MPa). The percentage deviations of his data are −2.5 to 3.3% above 350 °C, −1.0 to 1.2% between 200 and 300 °C, and −0.22 to 0.43% for a temperature of 100 °C except the following 9 points: 12.6%, 400 °C, 20 MPa; 12.9%, 400 °C, 25 MPa; 13.8%, 450 °C, 25 MPa; 9.1%, 450 °C, 35 MPa; 15.5%, 500 °C, 25 MPa; 7.4%, 500 °C, 50 MPa; 9.5%, 550 °C, 25 MPa; 6.8%, 550 °C, 50 MPa; 4.8%, 550 °C, 75 MPa. Fogo et al. [46] measured ϵ-data in the range of temperatures from 378 to 397 °C and of densities from 210 and 500 kg/m^3, and their data deviate within 0.35 in ϵ-units (8%). Lukashov et al. [63] measured ϵ-data in the range of temperatures from 400 to 600 °C and of densities from 80 to 400 kg/m^3, and their data deviate within 0.4 in ϵ-units (14%). The ϵ-value of Lukashov et al. at a condition of 500 °C and 240 kg/m^3 corresponding to a pressure of about 50 MPa is smaller than the calculated value by 8.2%, whereas Heger's value is larger than the calculated value by 9.5%. The ϵ-values of Lukashov et al. for a temperature of 600 °C are smaller than the ϵ-values calculated from the equation proposed by Onsager [69].

The data of Harris et al. [44,45] covering the range of temperatures from 14 to 75 °C up to 20 MPa deviate within 0.12 in ϵ-units (0.15%) between 14 and 36 °C, whereas deviations of their data for temperatures above 47 °C increase with temperature to a maximum value of 0.85 in ϵ-units (1.37%) at a condition of 75 °C and 3.8 MPa. The data of Kyropoulos [27], who measured only for a temperature of 20 °C with the claimed accuracy of 2% corresponding to 2 in ϵ-units, deviate by 0.5 to 2.7 in ϵ-units. Those of Scaife [47], who also published ϵ-data only for a temperature of 20 °C and claimed an accuracy of 4% corresponding to about 3.5 in ϵ-units, deviate by 0.9 to 5 in ϵ-units. The deviations in both cases increase with pressure. Although deviations of the data by Schadow and Steiner [57], who measured in the range of temperatures from 20 to 45 °C up to 130 MPa, do not exceed 0.4 in ϵ-units for two temperatures of 20 and 30 °C, the data for a temperature of 45 °C deviate by −0.3 to −1.1 in ϵ-units which is much larger than the claimed accuracy in their data of 0.05% corresponding to about 0.04 in ϵ-units.

A comparison for the static dielectric constant of water at atmospheric pressure in the range 0–100 °C is shown in figure 7. The calculated values agree with the experimental data [33,36,38,41,48–50,52,53,55,56,62] within 0.26 in ϵ-units corresponding to 0.3% between 0 and 75 °C. The deviations of the data by Malmberg and Maryott [48] for temperatures above 75 °C increase with temperature to a maximum value of 0.30 in ϵ-units (0.55%) at 100 °C, whereas those by Åkerlöf [36] decrease with increasing temperature to a maximum value of −0.32 in ϵ-units (−0.58%). Wyman's data [33] in the range 0–100 °C agree with the cal-

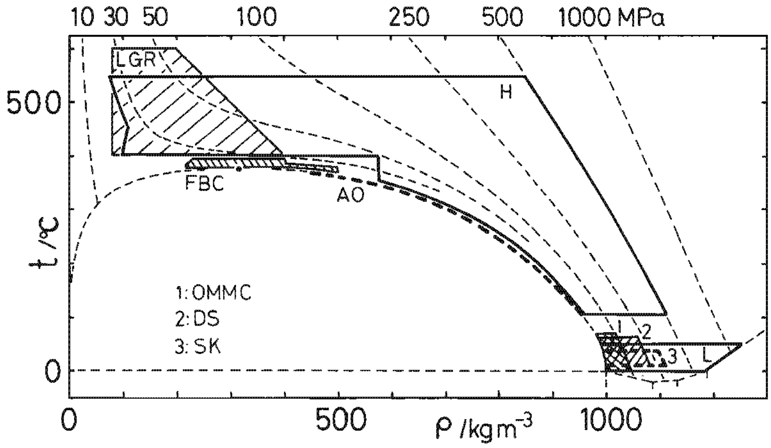

FIGURE 1. Ranges covered by the experimental data.

L:	Lees [42]	DS:	Dunn and Stokes [55]
AO:	Akerlof and Oshry [43]	H:	Heger [58]
FBC:	Fogo et al. [46]	SK:	Srinivasan and Kay [62]
OMMC:	Owen et al. [49]	LGR:	Lukashov et al. [63]

Table 2. Selected experimental data

Author	Year	Region	Evaluated weighting factor	Reference
Drake	(1930)	1	1	[31]
Devoto	(1930)	1	1	[32]
Wyman	(1930/38)	1	3	[33, 39]
Lattey	(1931)	1	1	[35]
Åkerlöf	(1932)	1	3	[36]
Albright	(1937)	1	3	[38]
Jones	(1939)	1	1	[40]
Albright	(1946)	1	5	[41]
Lees	(1949)	H	5	[42]
Akerlof	(1950)	S	5	[43]
Harris	(1953)	H	1	[44, 45]
Fogo	(1954)	H	3	[46]
Malmberg	(1956)	1	10	[48]
Owen	(1961)	1	10	[49]
Owen	(1961)	H	5	[49]
Kay	(1962/67/69)	1	5	[50, 53, 56]
Rusche	(1966)	1	5	[52]
Dunn	(1969)	1	10	[55]
Dunn	(1969)	H	5	[55]
Heger	(1969)	H	5	[58]
Kay	(1974)	1	10	[62]
Kay	(1974)	H	5	[62]
Lukashov	(1975)	H	5	[63]

Region:
1: water at atmospheric pressure, H: water and steam under high pressures, S: saturated water

Table 3. Numerical values of the coefficients in eq(1)

A_1	=	7.62571×10^0
A_2	=	2.44003×10^2
A_3	=	-1.40569×10^2
A_4	=	2.77841×10^1
A_5	=	-9.62805×10^1
A_6	=	4.17909×10^1
A_7	=	-1.02099×10^1
A_8	=	-4.52059×10^1
A_9	=	8.46395×10^1
A_{10}	=	-3.58644×10^1
T_0	=	298.15 K
ρ_0	=	1000 kg/m^3

Table 4. Static dielectric constant of water and steam

Units: t°C, P MPa

P	t						
	0.0	25.0	50.0	75.0	100.0	125.0	150.0
0.1	87.81	78.46	69.91	62.24	1.00	1.00	1.00
0.5	87.83	78.47	69.92	62.25	55.43	49.36	43.95
1.0	87.86	78.49	69.94	62.27	55.44	49.37	43.96
2.5	87.93	78.55	69.99	62.33	55.50	49.43	44.02
5.0	88.05	78.65	70.09	62.42	55.59	49.52	44.12
10.0	88.28	78.85	70.27	62.59	55.76	49.70	44.30
20.0	88.75	79.24	70.63	62.94	56.11	50.05	44.66
30.0	89.20	79.63	70.98	63.28	56.44	50.39	45.01
40.0	89.64	80.00	71.32	63.61	56.77	50.72	45.34
50.0	90.07	80.36	71.66	63.93	57.08	51.03	45.67
60.0	90.49	80.72	71.98	64.24	57.39	51.34	45.98
70.0	90.90	81.07	72.30	64.54	57.69	51.64	46.28
80.0	91.29	81.42	72.62	64.84	57.98	51.93	46.57
90.0	91.67	81.75	72.92	65.13	58.27	52.21	46.86
100.0	92.04	82.08	73.22	65.42	58.55	52.49	47.14
125.0	92.89	82.84	73.93	66.09	59.19	53.12	47.78
150.0	93.71	83.57	74.62	66.74	59.82	53.75	48.40
175.0	94.48	84.28	75.27	67.36	60.42	54.34	48.98
200.0	95.20	84.94	75.89	67.95	61.00	54.90	49.54
225.0	95.87	85.58	76.50	68.53	61.55	55.44	50.08
250.0	96.51	86.20	77.08	69.08	62.08	55.96	50.59
300.0	97.69	87.34	78.17	70.14	63.10	56.94	51.55
350.0	98.75	88.40	79.19	71.12	64.05	57.86	52.45
400.0	99.72	89.39	80.13	72.03	64.94	58.74	53.30
450.0	100.60	90.30	81.02	72.89	65.78	59.56	54.10
500.0	101.42	91.16	81.84	73.69	66.57	60.33	54.85

Table 4. -- Continued

Units: t°C, P MPa

P	t 175.0	200.0	225.0	250.0	275.0	300.0	350.0
0.1	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.5	1.01	1.01	1.01	1.01	1.01	1.01	1.01
1.0	39.11	1.02	1.02	1.02	1.02	1.02	1.01
2.5	39.17	34.79	1.07	1.06	1.05	1.04	1.04
5.0	39.28	34.90	30.89	27.15	1.13	1.11	1.08
10.0	39.47	35.11	31.13	27.43	23.90	20.39	1.23
20.0	39.85	35.52	31.58	27.95	24.54	21.24	14.07
30.0	40.22	35.91	32.01	28.43	25.11	21.95	15.66
40.0	40.56	36.28	32.40	28.87	25.61	22.56	16.72
50.0	40.89	36.63	32.78	29.28	26.08	23.10	17.55
60.0	41.21	36.96	33.13	29.67	26.50	23.58	18.24
70.0	41.52	37.28	33.47	30.03	26.90	24.02	18.84
80.0	41.82	37.59	33.79	30.37	27.27	24.43	19.37
90.0	42.11	37.89	34.10	30.70	27.62	24.81	19.85
100.0	42.39	38.17	34.40	31.01	27.95	25.17	20.29
125.0	43.05	38.86	35.13	31.78	28.76	26.03	21.26
150.0	43.68	39.50	35.78	32.46	29.47	26.77	22.09
175.0	44.27	40.10	36.39	33.09	30.12	27.45	22.83
200.0	44.83	40.66	36.97	33.67	30.72	28.07	23.49
225.0	45.36	41.20	37.51	34.22	31.28	28.64	24.09
250.0	45.87	41.70	38.02	34.74	31.81	29.17	24.65
300.0	46.82	42.65	38.97	35.69	32.77	30.15	25.65
350.0	47.70	43.52	39.83	36.56	33.64	31.02	26.53
400.0	48.53	44.33	40.64	37.36	34.43	31.81	27.32
450.0	49.31	45.10	41.38	38.09	35.16	32.54	28.04
500.0	50.05	45.82	42.09	38.78	35.84	33.21	28.70

Table 4. -- Continued

Units: t°C, P MPa

P	t			
	400.0	450.0	500.0	550.0
0.1	1.00	1.00	1.00	1.00
0.5	1.01	1.00	1.00	1.00
1.0	1.01	1.01	1.01	1.01
2.5	1.03	1.03	1.02	1.02
5.0	1.07	1.06	1.05	1.04
10.0	1.17	1.14	1.11	1.10
20.0	1.64	1.42	1.32	1.26
30.0	5.91	2.07	1.68	1.51
40.0	10.46	3.84	2.34	1.90
50.0	12.16	6.57	3.45	2.48
60.0	13.28	8.53	4.90	3.26
70.0	14.16	9.87	6.31	4.20
80.0	14.88	10.88	7.50	5.16
90.0	15.50	11.70	8.47	6.06
100.0	16.05	12.39	9.29	6.88
125.0	17.21	13.77	10.88	8.53
150.0	18.16	14.85	12.07	9.80
175.0	18.98	15.74	13.04	10.81
200.0	19.69	16.51	13.86	11.65
225.0	20.33	17.19	14.56	12.38
250.0	20.91	17.80	15.19	13.01
300.0	21.94	18.85	16.25	14.07
350.0	22.83	19.74	17.14	14.93
400.0	23.62	20.52	17.89	15.66
450.0	24.32	21.20	18.55	16.28
500.0	24.96	21.82	19.14	16.83

Table 5. Static dielectric constant of saturated water and steam

Units: t°C

t	ε_L	ε_V	t	ε_L	ε_V
0.0	87.81	1.00	200.0	34.74	1.04
10.0	83.99	1.00	210.0	33.11	1.05
20.0	80.27	1.00	220.0	31.53	1.06
30.0	76.67	1.00	230.0	30.01	1.07
40.0	73.22	1.00	240.0	28.53	1.09
50.0	69.90	1.00	250.0	27.08	1.11
60.0	66.73	1.00	260.0	25.68	1.13
70.0	63.70	1.00	270.0	24.30	1.15
80.0	60.81	1.00	280.0	22.94	1.18
90.0	58.05	1.00	290.0	21.60	1.22
100.0	55.41	1.00	300.0	20.26	1.27
110.0	52.90	1.01	310.0	18.92	1.33
120.0	50.50	1.01	320.0	17.56	1.40
130.0	48.22	1.01	330.0	16.17	1.50
140.0	46.03	1.01	340.0	14.72	1.64
150.0	43.94	1.01	350.0	13.16	1.85
160.0	41.95	1.02	360.0	11.36	2.19
170.0	40.03	1.02	370.0	8.70	3.00
180.0	38.20	1.03			
190.0	36.44	1.03			

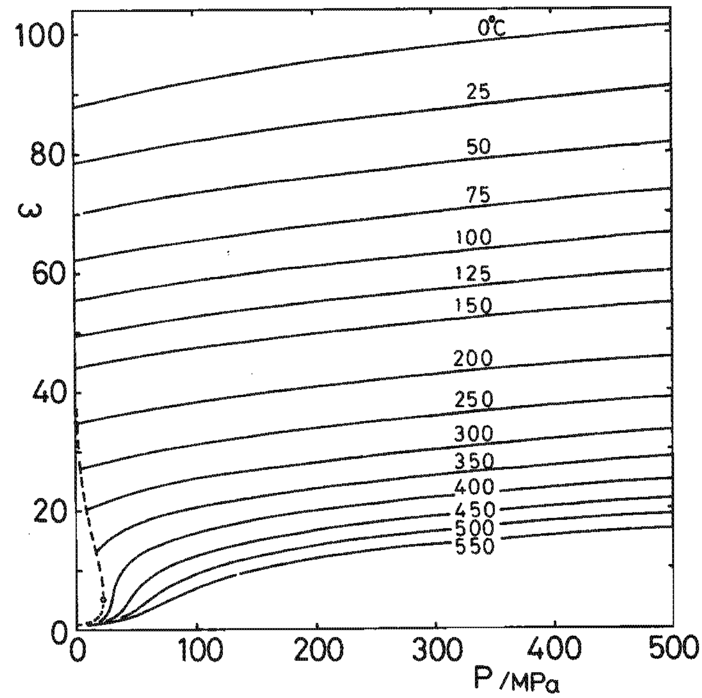

FIGURE 2. Static dielectric constant as a function of pressure.

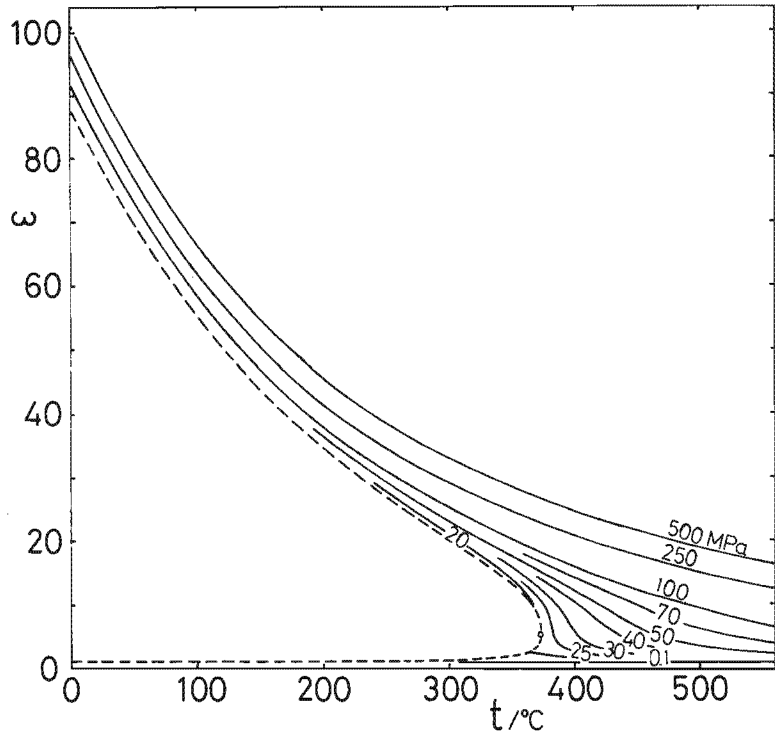

FIGURE 3. Static dielectric constant as a function of temperature.

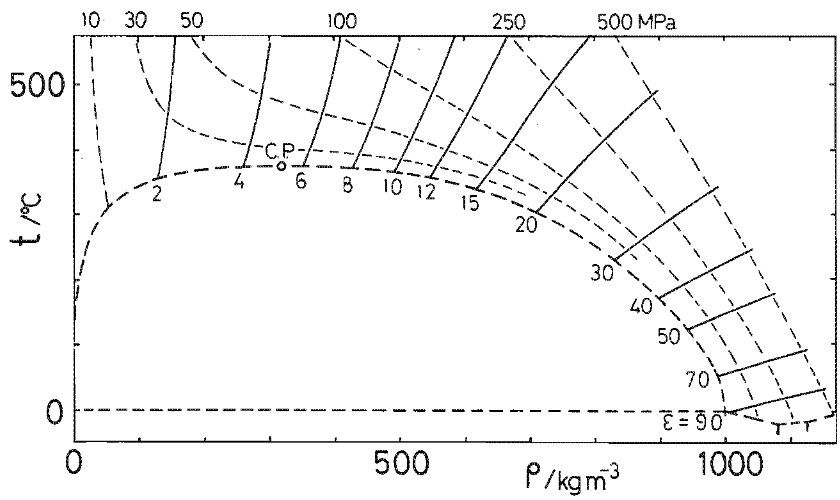

FIGURE 4. Curves of constant ϵ as a function of temperature and density.

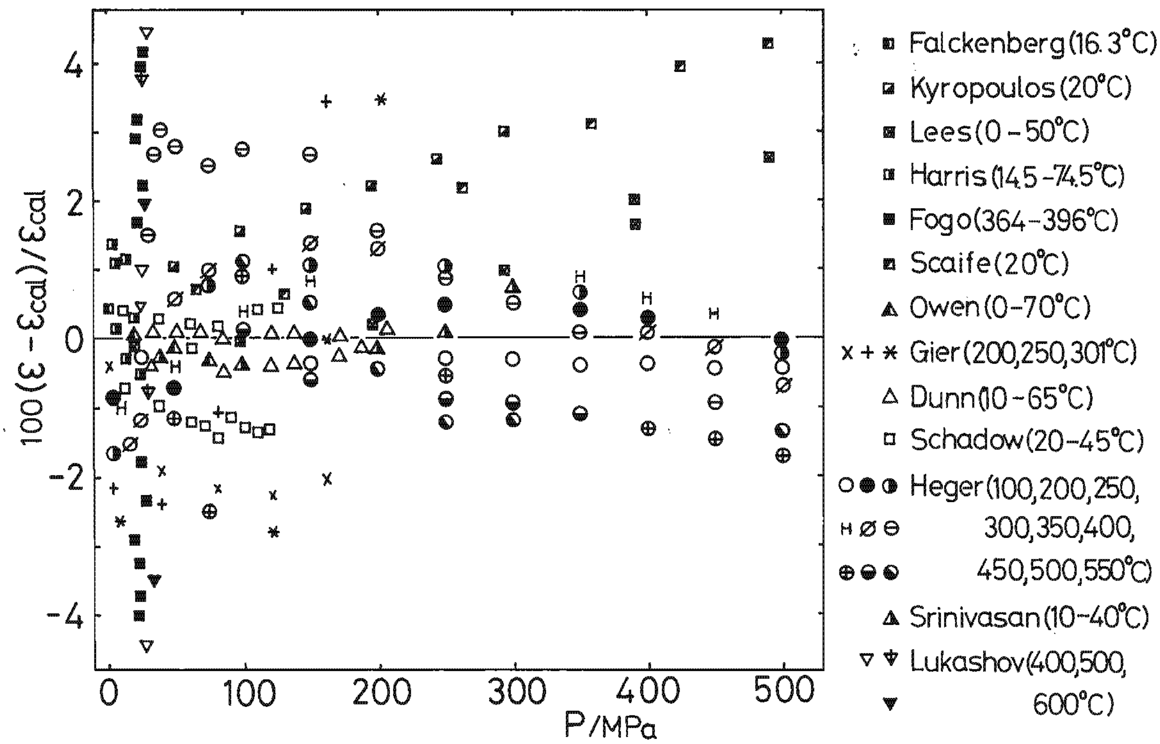

FIGURE 5. Deviation of experimental data from eq (1) as a function of pressure.

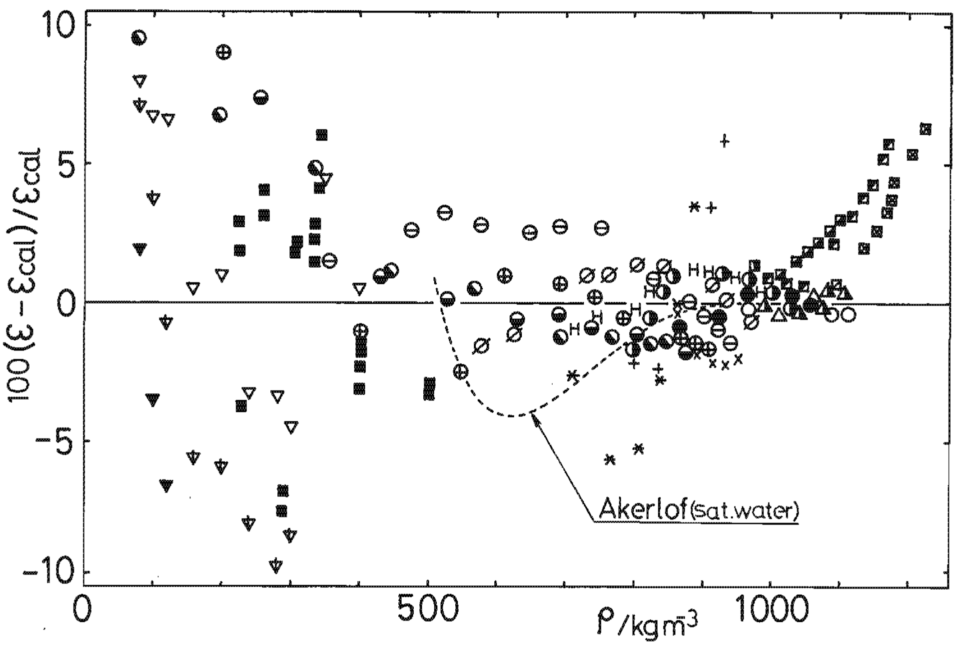

FIGURE 6. Deviation of experimental data from eq (1) as a function of density. See key to symbols for figure 5.

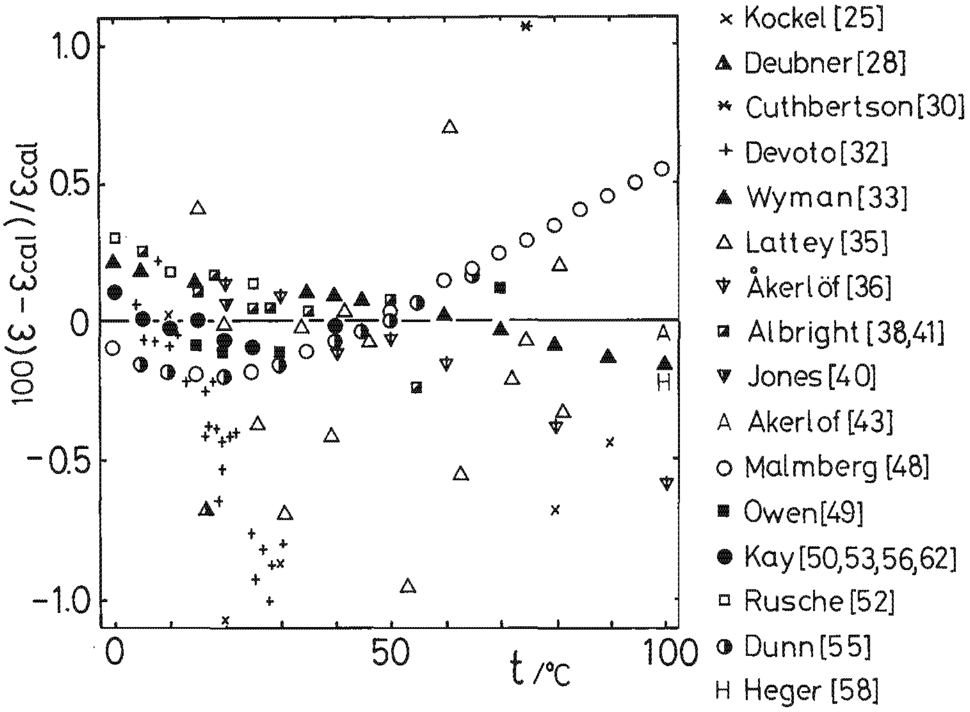

FIGURE 7. Deviation of experimental data at atmospheric pressure from eq (1).

culated values within 0.18 in ϵ-units (0.21%). The ϵ-values measured by Akerlof and Oshry [43] and by Heger [58] at 100 °C agree with -0.03 and -0.12 in ϵ-units (-0.05% and -0.22%), respectively.

3.4. Uncertainty of the New ICPS Representation

In view of the region covered by the experimental data, eq (1) is applicable in the region of temperatures between 0 and 550 °C up to a maximum pressure of 500 MPa. Based on the comparison with experimental data, the uncertainty of the ϵ-values calculated by eq (1) is estimated to be 0.5% for temperatures of $T \leqslant 100$ °C, to be 1% for temperatures of $100 < T \leqslant 250$ °C, to be 2.5% in the range of temperatures from 250 to 550 °C and of densities from 300 kg/m^3 to that corresponding to a maximum pressure of 500 MPa, and to be 5% for densities from 120 to 300 kg/m^3 where ϵ is not greater than 5. The ϵ-values calculated by eq (1) for $\epsilon < 1.8$ covering the density region of $\rho < 120$ kg/m^3 are smaller than the ϵ-values calculated from the equation proposed by Onsager [69], although the differences do not exceed 0.1 in ϵ-units. Therefore the uncertainty of the ϵ-values calculated by eq (1) in that region is estimated to be more than 5%. Examining curves of constant ϵ, eq (1) appears applicable in the extrapolated range of temperatures and pressures up to 600 °C and 1 GPa. In the range of temperatures above 600 °C, however, the behavior of curves of constant ϵ generated using eq (1) shows that the calculated values are unreliable. The estimated uncertainties are illustrated in figure 8.

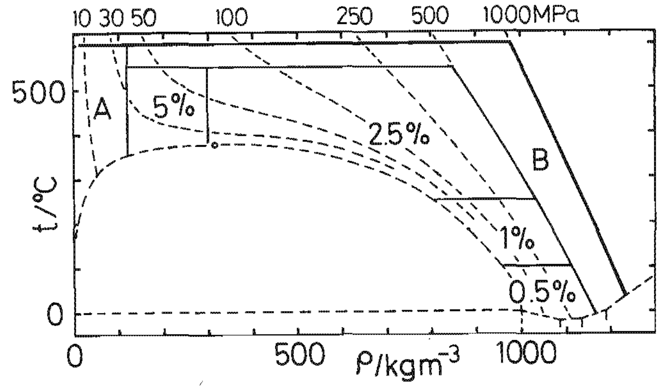

FIGURE 8. Uncertainty of the new ICPS Representation.
A: more than 5%.
B: can be extrapolated with reduced reliability.

3.5. Examination of Equations Proposed Previously

By using eq (1) as a reference standard, the applicability of equations previously proposed [12,13,70–72] can be examined. These equations are divided into two categories: an empirical expression based on the so-called "Kirkwood-correlation factor" [12,13,71] and an empirical correlation for ϵ like eq (1) [70,72]. The values by Franck's equation [12] agree with those from eq (1) within a deviation of 4% in the region of temperatures from 300 and 500 °C and of densities between 300 and 800 kg/m^3. The deviations of Franck's equation for densities of $\rho > 900$ kg/m^3 increase with density to a maximum of 20% at a condition of 500 °C and 1000 kg/m^3. The values from the equation of Quist and Marshall [13] agree with those from eq (1) within 4% in the range of temperatures from 0 to 500 °C and of densities between 300 and 1000 kg/m^3. The values from the Quist-Marshall equation at a density of 1000 kg/m^3 between 0 to 400 °C are in good agreement with those from eq (1) within 1%. The equation proposed by Lukashov [71] yields large deviations between −8 and 10% at temperatures between 300 and 500 °C and densities up to 1000 kg/m^3.

The equation of Helgeson and Kirkham [70] describes the experimental data as well as eq (1) in the range of densities larger than 400 kg/m^3. The curves of constant ϵ generated using the Helgeson-Kirkham equation show a certain amount of ripple on a temperature-density diagram, which is one of unacceptable characteristics for empirical correlations. This ripple increases with decreasing ϵ-values and that for $\epsilon < 30$ becomes a considerable amount. Helgeson-Kirkham equation can not be applicable in the range of temperatures above 550 °C and of densities smaller than 300 kg/m^3, respectively. The equation of Golubev et al. [72] describes the experimental data as well as eq (1) in the range of temperatures between 50 and 300 °C, although their equation has 7 coefficients only. However their equation does not reproduce the behavior of experimental ϵ-data in the high temperature region above 400 °C.

Acknowledgement

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Appendix

The Eighth International Conference on the Properties of Steam

Giens, France, September 1974

Release on Static Dielectric Constant of Water Substance

September 1977

Unrestricted publication allowed in all countries.

Issued by the International Association for the Properties of Steam.

President: Mr. H. W. Bradly
Bradly Associates
3 Belleville Drive
Oadby, Leicester LE2 4HA, U.K.

Executive Secretary: Dr. Howard J. White, Jr.
Office of Standard Reference Data
National Bureau of Standards
Washington, D.C. 20234, U.S.A.

The Eighth International Conference on the Properties of Steam designated a Working Group III, consisting of representatives of France, the Federal Republic of Germany, Japan, the USA, and the USSR, for the purpose of establishing representations of miscellaneous properties.

Working Group III met in Schliersee near Munich in April 1975, in Ottawa in September 1975, in Kyoto in September 1976, and in Moscow in September 1977, and completed its work with respect to the representation of the static dielectric constant (permittivity), ϵ, which is valid for electric fields of very low frequency and moderate intensity, of water substance.

In accordance with a resolution of the Eighth Conference, the material included in the present release was circulated to and approved by the Heads of all National Delegations attending the Eighth Conference (Canada, Czechoslovakia, Federal Republic of Germany, France, Hong Kong, Hungary, Japan, Netherlands, Poland, Switzerland, United Kingdom, United States of America, and the Union of Soviet Socialist Republics).

This Release on static dielectric constant is now issued by the Secretariat under the full authority of the Eighth Conference, and presents in the accompanying Appendices the *International Representation of the Static Dielectric Constant of Water Substance, 1977.* For further information write to

Dr. Howard J. White, Jr.
Executive Secretary, IAPS
Office of Standard Reference Data
Physics Building, Room A316
National Bureau of Standards
Washington, D.C. 20234 USA

Attachments

Appendices A, B, and C.

Appendix A

Working Group III has collected the existing experimental data from the literature, which were included in the document, "Survey of the Experimental Studies of the Static Dielectric Constant of Water," by E. U. Franck, W. Harder, and W. Hill, presented to the meeting of Working Group III of the International Association for the Properties of Steam (IAPS), Schliersee, near Munich, Federal Republic of Germany, April 1975. Working Group III considers that these data are not sufficiently accurate and precise to allow definition of a two-dimensional representation that satisfies all of the criteria for smoothness and physical plausibility that logically can be required of it. Working Group III hopes that additional measurements of superior quality will become available in the future. At the present time, in this release, Working Group III issues a formulation consisting of an equation and one table. This equation is considered to be as good as possible a representation of these available data.

Appendix B

Recommended Interpolation Equation

The available experimental data are reproduced with a standard deviation of 0.33 in ϵ–units by the use of the formula given below, wherein

ϵ : static dielectric constant

ρ : density in kg/m^3 [†]

T : temperature in K on the 1968 Practical Temperature Scale

$T^* = T / T_0$

$\rho^* = \rho / \rho_0$

[†] For preference and to reproduce the values given in Appendix C, the density should be computed with the aid of the 1968 IFC Formulation for Scientific and General Use for pressures up to 100 MPa and with the equation of state proposed by Juza (Roz. Cesk. Akad. Ved Rada Tech. Ved, 1966, vol. 76, No. 1) for pressures above 100 MPa.

a_i, T_0, ρ_0 : numerical constants

$$\begin{aligned}\epsilon = 1 \quad & + (a_1 / T^*) \rho^* \\ & + (a_2 / T^* + a_3 + a_4 T^*) \rho^{*2} \\ & + (a_5 / T^* + a_6 T^* + a_7 T^{*2}) \rho^{*3} \\ & + (a_8 / T^{*2} + a_9 / T^* + a_{10}) \rho^{*4}\end{aligned}$$

The constants appearing in the preceding equation have the numerical values given below:

$$\begin{aligned}
a_1 &= 7.625\,71 \times 10^0 \\
a_2 &= 2.440\,03 \times 10^2 \\
a_3 &= -1.405\,69 \times 10^2 \\
a_4 &= 2.778\,41 \times 10^1 \\
a_5 &= -9.628\,05 \times 10^1 \\
a_6 &= 4.179\,09 \times 10^1 \\
a_7 &= -1.020\,99 \times 10^1 \\
a_8 &= -4.520\,59 \times 10^1 \\
a_9 &= 8.463\,95 \times 10^1 \\
a_{10} &= -3.586\,44 \times 10^1 \\
T_0 &= 298.15 \text{ K} \\
\rho_0 &= 1000 \text{ kg/m}^3
\end{aligned}$$

This equation is valid in the range

$$273.15 \text{ K} \leqslant T \leqslant 823.15 \text{ K}$$

in temperature, and

$$0 \leqslant \rho \leqslant 1150 \text{ kg/m}^3$$

in density, which corresponds to an approximate pressure range

$$0 \leqslant P \leqslant 500 \text{ MPa}.$$

A discussion of the equation and its derivation is given in the document, "The Static Dielectric Constant of Water in the Range of Temperatures from 0 to 550 °C and Pressures up to 5 kbar," by M. Uematsu, W. Harder, and E. U. Franck, presented to the meeting of Working Group III of IAPS, Kyoto, Japan, September 1976; and also in a presentation, "The Equation of State for the Static Dielectric Constant of Water in the Range of Temperatures from 0 to 550 °C and Pressures up to 5 kbar," by M. Uematsu and E. U. Franck, in the Proceedings of the 6th AIRAPT International High Pressure Conference, Boulder, Colorado, U.S.A., July 1977.

Appendix C

Static Dielectric Constant of Water Substance

A table of values is given at selected points of pressure P in MPa and temperature T in K obtained from the equation given in Appendix B.

Table Static Dielectric Constant of Water and Steam

P/T	273.15	298.15	323.15	348.15	373.15	398.15	423.15	448.15	473.15	498.15	523.15	548.15	573.15	623.15	673.15	723.15	773.15	823.15
10	88.28	78.85	70.27	62.59	55.76	49.70	44.30	39.47	35.11	31.13	27.43	23.90	20.39	1.23	1.17	1.14	1.11	1.10
20	88.75	79.24	70.63	62.94	56.11	50.05	44.66	39.85	35.52	31.58	27.95	24.54	21.24	14.07	1.64	1.42	1.32	1.26
30	89.20	79.63	70.98	63.28	56.44	50.39	45.01	40.22	35.91	32.01	28.43	25.11	21.95	15.66	5.91	2.07	1.68	1.51
40	89.64	80.00	71.32	63.61	56.77	50.72	45.34	40.56	36.28	32.40	28.87	25.61	22.56	16.72	10.46	3.84	2.34	1.90
50	90.07	80.36	71.66	63.93	57.08	51.03	45.67	40.89	36.63	32.78	29.28	26.08	23.10	17.55	12.16	6.57	3.45	2.48
60	90.49	80.72	71.98	64.24	57.39	51.34	45.98	41.21	36.96	33.13	29.67	26.50	23.58	18.24	13.28	8.53	4.90	3.26
70	90.90	81.07	72.30	64.54	57.69	51.64	46.28	41.52	37.28	33.47	30.03	26.90	24.02	18.84	14.16	9.87	6.31	4.20
80	91.29	81.42	72.62	64.84	57.98	51.93	46.57	41.82	37.59	33.79	30.37	27.27	24.43	19.37	14.88	10.88	7.50	5.16
90	91.67	81.75	72.92	65.13	58.27	52.21	46.86	42.11	37.89	34.10	30.70	27.62	24.81	19.85	15.50	11.70	8.47	6.06
100	92.04	82.08	73.22	65.42	58.55	52.49	47.14	42.39	38.17	34.40	31.01	27.95	25.17	20.29	16.05	12.39	9.29	6.88
125	92.89	82.84	73.93	66.09	59.19	53.12	47.78	43.05	38.86	35.13	31.78	28.76	26.03	21.26	17.21	13.77	10.88	8.53
150	93.71	83.57	74.62	66.74	59.82	53.75	48.40	43.68	39.50	35.78	32.46	29.47	26.77	22.09	18.16	14.85	12.07	9.80
175	94.48	84.28	75.27	67.36	60.42	54.34	48.98	44.27	40.10	36.39	33.09	30.12	27.45	22.83	18.98	15.74	13.04	10.81
200	95.20	84.94	75.89	67.95	61.00	54.90	49.54	44.83	40.66	36.97	33.67	30.72	28.07	23.49	19.69	16.51	13.86	11.65
225	95.87	85.58	76.50	68.53	61.55	55.44	50.08	45.36	41.20	37.51	34.22	31.28	28.64	24.09	20.33	17.19	14.56	12.38
250	96.51	86.20	77.08	69.08	62.08	55.96	50.59	45.87	41.70	38.02	34.74	31.81	29.17	24.65	20.91	17.80	15.19	13.01
300	97.69	87.34	78.17	70.14	63.10	56.94	51.55	46.82	42.65	38.97	35.69	32.77	30.15	25.65	21.94	18.85	16.25	14.07
350	98.75	88.40	79.19	71.12	64.05	57.86	52.45	47.70	43.52	39.83	36.56	33.64	31.02	26.53	22.83	19.74	17.14	14.93
400	99.72	89.39	80.13	72.03	64.94	58.74	53.30	48.53	44.33	40.64	37.36	34.43	31.81	27.32	23.62	20.52	17.89	15.66
450	100.60	90.30	81.02	72.89	65.78	59.56	54.10	49.31	45.10	41.38	38.09	35.16	32.54	28.04	24.32	21.20	18.55	16.28
500	101.42	91.16	81.84	73.69	66.57	60.33	54.85	50.05	45.82	42.09	38.78	35.84	33.21	28.70	24.96	21.82	19.14	16.83