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Thermodynamic Properties of Steam in the Critical Region

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An analysis is presented of the experimental data on thermodynamic properties in the critical region of steam. The model used is that of revised and extended scaling, as given by the modern theory of critical phenomena. All thermodynamic properties are given in closed (parametric) form. The model has, in addition to three universal constants that are given by theory, sixteen adjustable parameters that were obtained by least-squares fit to *PVT* and speed-of-sound data. It is valid in the range 200–420 kg/m^3 in density and 644–693 K in temperature. It accurately represents the experimental data for equation of state, vapor pressure, latent heat, specific heats C_p and C_v and speed of sound. Our analysis permits new estimates of the critical parameters of steam, and has led to a number of conclusions regarding the mutual consistency of the experimental data. Tabulated values of the thermodynamic properties of steam are appended to the paper.

Key words: critical parameters; critical region; energy; equation of state; latent heat; scaling laws; specific heat; speed of sound; steam; thermodynamic function; thermodynamic tables; vapor pressure; water.

Contents

1. Introduction

The modern theory of critical phenomena gives detailed predictions for the singular behavior of thermodynamic properties in fluids. *In this paper we present the results of an* analysis of the thermodynamic behavior of steam in the criti-

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cal region in terms of a revised and extended scaled fundamental equation that incorporates these theoretical predictions. The use of this equation makes it possible to assess the consistency between the data for various thermodynamic properties of steam, such as the *PVT* properties, specific heats, and speed of sound. Detailed comparisons of these experimental data and the theoretical surface are presented. Tabulated values of the thermodynamic properties pressure, enthalpy, entropy, energy, specific heats at constant pressure and at constant volume, and speed of sound are given with temperature and density as entries. These tables will be helpful for testing global formulations of the thermodynamic behavior of steam for adequacy in the critical region, and for supplementing such formulations if they are found wanting.

In this paper, we introduce the concepts of critical anomalies and critical exponents in Sec. 2. The fundamental equation is developed in Sec. 3, while a complete listing of the equations for the thermodynamic properties is given in Appendix A, and the constants defining the equation are listed in Appendix B. In Sec. 4 the various data sources are discussed. Section 5 deals with the selection of critical constants for steam. Sections 6–10 are devoted to comparisons of experimental data for the equation of state, vapor pressure, latent heat, energy, specific heats C_v and C_p, and speed-of-sound with the predictions of our surface. Section 11 gives the values of the critical amplitude ratios, and compares them with the universal theoretical values. As mentioned in Sec. 12, tables of the thermodynamic properties of steam in the critical region have been prepared. These tables of the thermodynamic properties as a function of temperature along isochores, and also as a function of temperature and of pressure at saturation, are given in Appendix C. Appendix D lists the computer program used to generate the tables of thermodynamic properties. Appendix E gives units and conversion factors. Section 13 summarizes the principal conclusions of our work.

2. Critical Exponents

It is well known that thermodynamic properties show anomalous behavior when the critical point is approached. These critical anomalies are described by so-called power laws. If we denote by the symbol Δ the departure of a property from its critical value, and denote by an asterisk (*) a property value made dimensionless by appropriate combinations of the critical pressure, temperature, and density, then examples of these power laws are the following[1]:

Compressibility along critical isochore: $K_T^* = \Gamma^{\pm} |\Delta T^*|^{-\gamma}$,

Pressure along critical isotherm: $|\Delta P^*| = D\,|\Delta\rho^*|^{\delta}$,

Density along coexistence curve: $|\Delta\rho^*| = B\,|\Delta T^*|^{\beta}$,

Specific heat at constant volume along critical isochore: $C_v^* = A^{\pm} |\Delta T^*|^{-\alpha}$. (1)

Here ρ denotes the density, T the temperature, P the pressure. The superscript $^+$ denotes an approach to the critical temperature from higher, the superscript $^-$ the same from lower temperatures. Only two of the critical exponents α, β, γ, and δ are independent. The others follow from them through scaling relations listed in Appendix A, (A.13) and (A.14). The constants $\Gamma^{\pm}, D, B, A^{\pm}$ are called critical amplitudes.

The theory of critical phenomena asserts that the critical exponents of all fluids are the same, and to be identified with those calculated theoretically for three-dimensional Ising-like systems. This prediction has been verified by accurate experiments in a number of fluids. For reviews, we refer to Refs. 2–5. The available experimental material strongly supports the critical behavior of steam to be of the same type as that in other simpler fluids.[6] In this work, we have therefore fixed the critical exponents at values predicted theoretically, namely,[7]

$$\beta = 0.325, \quad \delta = 4.82, \quad \Delta_1 = 0.50, \tag{2}$$

where Δ_1 is a "gap" exponent to be introduced in the next section. Values of the other exponents then follow from relations (A.13) and (A.14).

3. Fundamental Equation

Our fundamental equation is the one introduced by Ley-Koo and Green[3,8]; these authors adapted, for use in representing fluid behavior, an expansion of the free energy around the critical point that had been proposed by Wegner[9] for magnetic systems. The potential used in our work is $\tilde{P} = P^*/T^*$ as a function of $\tilde{\mu} = \mu^*/T^*$ and $\tilde{T} = -1/T^*$. Here P is the pressure, μ the chemical potential, and T the absolute temperature. Asterisks denote reduced properties: $P^* = P/P_c$, $T^* = T/T_c$, and $\mu^* = \mu\rho_c/P_c$, where P_c is the critical pressure, T_c the critical temperature, and ρ_c the critical density. This potential differs from those used in engineering applications in that both dependent and independent variables are intensive or "field" variables. As a consequence, the thermodynamic surface has a very simple structure, the vapor and liquid parts meeting with a discontinuous change in slope along a seam below T_c. This structure is consistent with the facts that intensive ("field") properties are identical in coexisting phases while their derivatives

$$(\partial\tilde{P}/\partial\tilde{\mu})_T = \tilde{\rho}, \quad (\partial\tilde{P}/\partial\tilde{T})_{\tilde{\mu}} = \tilde{U} \tag{3}$$

change discontinuously. Here $\tilde{U} = U/P_c V$, the reduced energy density, and $\tilde{\rho} = \rho/\rho_c$. In an approach to the critical point, the discontinuity in slope diminishes, to disappear at T_c. At the same time, however, the curvature of the surface at the phase boundary increases, and at the critical point the second derivatives

$$(\partial^2\tilde{P}/\partial\tilde{\mu}^2)_{\tilde{T}} = \tilde{\chi}_T \tag{4}$$

as well as $(\partial^2\tilde{P}/\partial T^2)_{\tilde{\mu}}$ and $(\partial^2\tilde{P}/\partial\tilde{\mu}\partial\tilde{T})$ diverge strongly. Here $\tilde{\chi}_T$ denotes the reduced value of $\rho^2 T K_T$, K_T being the isothermal compressibility.

On the surface, there is only one direction singled out as special, namely, that of the seam, the vapor–liquid equilibrium curve. One can imagine this seam extrapolated into the one-phase region. We represent the seam by the relation between μ and T of its projection on the μ–T plane in terms of an analytic representation

$$\tilde{\mu}_0(\tilde{T}) = \tilde{\mu}_c + \sum_{j=1}^{3} \tilde{\mu}_j (\Delta \tilde{T})^j, \tag{5}$$

where

$$\Delta \tilde{T} = \tilde{T} + 1 . \tag{6}$$

The assumption of analyticity of $\tilde{\mu}_0(\tilde{T})$ is essential; it is hard to prove experimentally, although some evidence has been accumulated, mainly from the behavior of the two-phase C_v.[4,10] The analyticity is a feature not only of the Ising model and the lattice gas, but also of the decorated lattice models[11,12] that closely represent the asymmetry between the liquid and the vapor observed in the laboratory and that is reflected, for instance, in the temperature variation of the average of coexisting liquid and vapor densities ("slope of the diameter").

In scaling, the physical variables $\tilde{\mu}$, $\tilde{T}$ are transformed to a new coordinate system, one axis of which is the curve $\tilde{\mu}_0(\tilde{T})$. The other axis intersects the first at the critical point in a direction that is, in principle, arbitrary. Early scaling equations used the line $\tilde{T} = -1$, or $T = T_c$, as this axis. In the present work, we allow this axis to be in an arbitrary direction. This is achieved by writing the second variable as a linear combination of $\tilde{\mu} - \tilde{\mu}_0(\tilde{T}) = \Delta\tilde{\mu}$ and $\tilde{T} + 1 = \Delta\tilde{T}$. Thus, the independent scaling variables u_μ, u_t are constructed as follows, as analytic functions of $\tilde{\mu}$ and $\tilde{T}$:

$$\begin{aligned} a u_\mu &= \tilde{\mu} - \tilde{\mu}_0(\tilde{T}) = \Delta\tilde{\mu}, \\ u_t &= \Delta\tilde{T} + c\Delta\tilde{\mu}. \end{aligned} \tag{7}$$

Here a is an arbitrary scaling constant (see below) and c is the so-called mixing parameter.[3] The potential $\tilde{P}$ is now decomposed into two parts, one part $\tilde{P}_{\text{reg}}$, that is regular in the scaling variables and another part $\Delta\tilde{P}$, that contains the critical anomalies. One can think of the regular part as an analytic surface in $\tilde{P}(u_\mu, u_t)$ or $P(\mu, T)$ space, tangent to the thermodynamic surface at the critical point. It contains, therefore, the information on the critical-point values of the first derivatives ρ_c and U_c. The scaled part is to produce the infinities in second derivatives at the critical point. The asymptotic critical behavior is incorporated by postulating for this anomalous part the following generalized homogeneous form[1,13]:

$$\Delta\tilde{P} = a k_0 |u_t|^{2-\alpha} g_0\left(\frac{u_\mu}{|u_t|^{\beta\delta}}\right). \tag{8}$$

Here k_0 is a second arbitrary scaling constant; α, β, and δ are critical exponents defined in Eq. (1). Only two of these are independent; their values are universal, that is, the same for all fluids and other Ising-like systems. The function g_0 of the scaling fields is likewise universal, the argument in parentheses indicating functional dependence.

Apart from the implicit presence of the mixing parameter c, which disrupts symmetry in a nonuniversal manner, the scaled part of the potential contains only two arbitrary constants, a and k_0, which one can think of as setting two scales, namely, for two of the three physical variables. The relation of generalized homogeneity (8) does not allow a free choice of more than two such factors.

Wegner[9] showed that higher-order confluent singularities must be present at the critical point. In the present work, we will take one Wegner correction to scaling into account. Our potential thus has the form[3]:

$$\tilde{P}(u_t, u_\mu) = \tilde{P}_{\text{reg}}(u_t, u_\mu) + a k_0 |u_t|^{2-\alpha} g_0\left(\frac{u_\mu}{|u_t|^{\beta\delta}}\right) + a k_1 |u_t|^{2-\alpha+\Delta_1} g_1\left(\frac{u_\mu}{|u_t|^{\beta\delta}}\right). \tag{9}$$

The correction-to-scaling function g_1 is again a universal function of the same scaling variable $u_\mu / |u_t|^{\beta\delta}$. This function is multiplied by a different power of u_t than in the asymptotic term; a new critical exponent Δ_1 has been introduced, and, in addition, a new nonuniversal amplitude factor k_1.

For the regular part of the potential, we have chosen the form:

$$\begin{aligned} \tilde{P}_{\text{reg}} &= \tilde{P}_0(\tilde{T}) + \Delta\tilde{\mu} + \tilde{P}_{11}\Delta\tilde{\mu}\Delta\tilde{T}, \\ \tilde{P}_0(\tilde{T}) &= 1 + \sum_{j=1}^{3} \tilde{P}_j (\Delta\tilde{T})^j. \end{aligned} \tag{10}$$

It may be worth pointing out that the term $\tilde{P}_{11}\Delta\tilde{\mu}\Delta\tilde{T}$ gives a contribution to the density linear in temperature and thus is responsible, in part, for introducing asymmetry in our model. It would be dangerous to identify this contribution with the temperature dependence of the coexistence curve diameter, since a second important, but anomalous, contribution to the slope of the diameter, proportional to $|\Delta\tilde{T}|^{1-\alpha}$, comes from the "mixing of variables."

Having thus completed the formal presentation of our potential, we proceed to the computational part. An important device for handling scaled equations is the transformation to Schofield's[14] parametric variables r and θ, which denote, respectively, a distance from the critical point and a location on a contour of constant r. The scaling variables are transformed to parametric variables by the relations

$$\begin{aligned} u_\mu &= r^{\beta\delta}\theta(1-\theta^2), \\ u_t &= r(1 - b^2\theta^2), \end{aligned} \tag{11}$$

in which the constant b^2 is universal. The variable θ reaches the values ± 1 on the coexistence curve. In asymptotic scaling, the assumption is made that the density $\Delta\tilde{\rho}$ varies linearly in θ:

$$\Delta\tilde{\rho} = k_0 r^\beta \theta. \tag{12}$$

With this "linear-model" assumption, the potential and all its derivatives can be written in algebraically closed form, as simple polynomials in θ, with universal coefficients that are functions of the critical exponents β and δ.[1] We have made this assumption not only for the leading contribution to $\Delta\tilde{\rho}$, but also for that arising from the first Wegner correction. This choice leads to a similar set of polynomials in θ for the Wegner correction-to-scaling, with universal coefficients that are functions of the critical exponents β, δ, Δ_1. The relevant expressions are all listed in Appendix A. We stress here that the seemingly complicating device of transformation to parametric variables has considerable advantages. Scaled algebraically closed forms that have been postulated for the equation of state[10,15–17] are not integrable in closed form and other thermodynamic properties have to be derived from them by numerical integration. These forms invariably suffer from the additional defect that they have a line of higher-order nonanalyticities stretching out from the critical point into the one-phase region. The linear-model parametric form possesses neither of these defects. Moreover, the use of the parametric form greatly simplifies the Legendre transformations that the thermodynamicist cannot escape.

Our fundamental equation contains the following constants:

3 critical exponents (universal)	:	β, δ, Δ_1,
3 critical parameters	:	P_c, ρ_c, T_c,
5 parameters of the scaling function	:	a, k_0, k_1, c, b^2,
4 background parameters for the pressure	:	$\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \tilde{P}_{11}$,
4 background parameters for caloric properties	:	$\tilde{\mu}_c, \tilde{\mu}_1, \tilde{\mu}_2, \tilde{\mu}_3$.

In what follows we will discuss how these parameters were determined. The values of the constants that we have used are listed in Appendix B. The equation is valid in a range of temperatures and densities as indicated in Fig. 1. All temperatures quoted are on the International Practical Temperature Scale of 1968, IPTS 1968,[18] unless otherwise indicated. This potential was applied before by us to steam.[19] The present results differ from those previously reported principally in a modified choice of the value of T_c.

Finally, a few remarks about the critical amplitudes introduced in Eq. (1). These amplitudes refer to limiting behavior and therefore contain no contributions from the correction-to-scaling term. Since the leading asymptotic term contains only two free constants, the six critical amplitudes introduced in Eq. (1) cannot be independent. Four relations between these amplitudes are listed in the Appendix, Eqs. (A38)–(A41). Theoretical predictions exist for these universal amplitude ratios.[20] In our fits of the potential to the properties of steam, we have left the constant b^2 adjustable, so that a check on the universality of the amplitude ratios became possible.

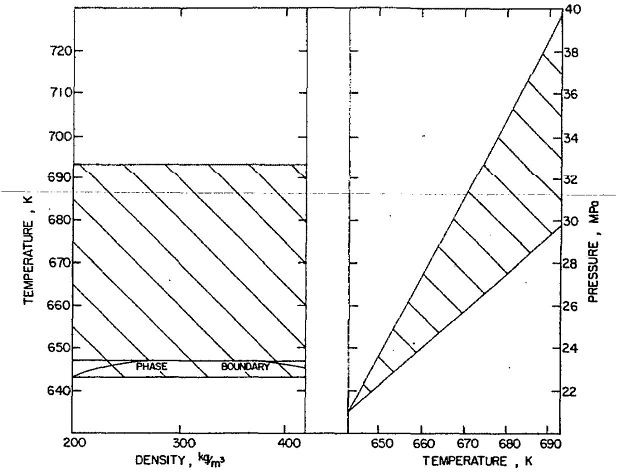

FIG. 1. Range of validity of the thermodynamic surface presented here in terms of (a) temperatures and densities, and (b) temperatures and pressures.

4. Data Sources for Steam

In Table 1 we list the data sources relevant to this paper. The vapor pressure[21] and latent heat[22] data of Osborne and collaborators are, although almost 50 yr old, of unsurpassed quality and indispensible in the formulation of the thermodynamic behavior of steam. We have used these data to validate our surface below the critical point and to determine the critical parameters of steam. The *PVT* data of Rivkin *et al.*[23,24] have formed the basis for determining most of the adjustable constants in our equation after we ascertained their consistency with the data of Osborne *et al.* The more recent high-quality *PVT* data of Kell *et al.*[25] are not in our range of interest, but we have ascertained the mutual consistency of the Kell and Osborne vapor pressure at 350 °C to be of the order of 5 mK. Caloric data of high quality are the C_p data of Sirota and Maltsev[26] and the speed-of-sound data of Erokhin and Kalyanov.[27] The latter were used to determine a few of the caloric "background" parameters in our equation. For both C_p and speed of sound, small adjustments of the temperature scales with respect to that of Rivkin *et al.* were required in order to obtain mutual consistency. The resulting thermodynamic surface, determined up to the integration constants for energy and entropy, was proven to be consistent with the energy and C_v data of Baehr and Schomäcker[28,29] and with selected C_v data of Amirkhanov *et al.*[30]

Table 1. Data sources

Property	First author	Ref	Year
Vapor pressure	Osborne	21	1933
Latent heat	Osborne	22	1937
PVT	Rivkin	23,24	1962-66
Specific heat C_p	Sirota	26	1962
Specific heat C_v and energy	Baehr	28,29	1974-75
Specific heat C_v	Amirkhanov	30	1974
Speed of sound	Erokhin	27	1980
P_c, T_c	Blank	31	1969
P_c, T_c, vapor pressure	Scheffler	32,33	1981

We have paid attention to the earlier[31] and to the recent direct determination of the critical parameters P_c, T_c of steam by Scheffler *et al.*,[32,33] and have found the latter marginally compatible with the surface presented here.

5. Critical Parameters for Steam

The values of the critical parameters P_c, T_c have been repeatedly determined in an apparatus first designed and built by Blank, an optical high-pressure cell in which the difference in refractive index of coexisting phases could be observed to disappear. In the course of the years, several

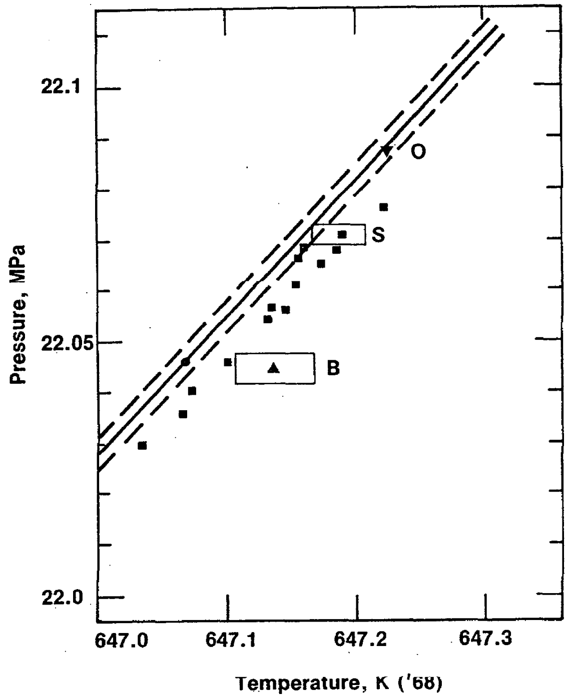

FIG. 2. The full curve is the vapor pressure curve and critical isochore according to our surface based on the *PVT* data of Rivkin *et al.* (Refs. 23 and 24). The dashed curves are confidence limits based on Rivkin's error estimates. The predicted vapor pressure curve agrees closely with the vapor pressures of Osborne *et al.* (Ref. 21). (Osborne's highest point, ▼, we believe to be slightly supercritical). The direct determinations of P_c, T_c are those of Blank, ▲, (Ref. 31) and of Scheffler *et al.*, ■, (Refs. 32 and 33). Their respective error estimates are indicated by rectangles. Vapor pressure measurements of Scheffler *et al.* ■, are also shown, as far as they are in the range of this plot. Our indirect determination of T_c based on the latent heats of Osborne *et al.* (Ref. 22) is indicated by ●; it is 0.12 K below the T_c value of Scheffler.

values of T_c were reported that ranged from 647.14 K[31] to 647.19 K in the most recent experiment by Scheffler *et al.*[32,33] The value of P_c reported by Blank falls about 0.01 MPa below the surface determined by Rivkin's data (Fig. 2). Scheffler *et al.* made considerable improvement in the pressure measurements; they reported $T_c = (647.19 \pm 0.02)$ K, $P_c = (22.071 \pm 0.002)$ MPa.

Although this point lies about 0.006 MPa below Rivkin's *PVT* surface (Fig. 2), this pressure difference is within the combined estimated uncertainty. We have determined critical-point parameters in a different way, namely, by analysis of appropriate fluid property data in the critical region. The analysis tools are the scaling laws, and the analysis proceeds in two steps. First, from experimental values of extensive properties in coexisting phases, such as densities, refractive indices, or latent heats, the temperature at which these properties become equal, i.e., the critical temperature T_c, can be estimated with a reliability depending on the accuracy and distribution of the property data considered. We were fortunate to have the latent heat data of Osborne *et al.* available for this purpose. Secondly, from a scaled analysis of vapor pressure and *PVT* data the vapor pressure curve and its extension, the critical isochore, can be established with good precision, but the actual value of T_c is usually poorly defined. Once, however, the value of T_c is established by other means, that of P_c follows immediately. This is the path we have followed in an indirect determination of the critical parameters of steam. The vapor pressure and latent heat data of Osborne *et al.*[21,22] have documented traceability and accuracy of pressure and temperature scales, and are consistent with Rivkin's *PVT* data to a better degree than with the (P_c, T_c) value of Scheffler *et al.* Therefore we have refrained from using the latter.

Osborne *et al.* measured the latent heats *g*, *b*, accompanying isothermal extraction of unit mass of vapor, resp. liquid, from their calorimeter. These quantities are related to the densities ρ_L, ρ_V of the phases and the vapor pressure by

$$
\begin{aligned}
b &= (T/\rho_L)(\mathrm{d}P/\mathrm{d}T)_{\mathrm{vap}}, \\
g &= (T/\rho_V)(\mathrm{d}P/\mathrm{d}T)_{\mathrm{vap}}.
\end{aligned}
\tag{13}
$$

The known scaled expressions for ρ_L, ρ_V, and $\mathrm{d}P/\mathrm{d}T$ (A32, A33) imply those for *b* and *g*. To leading order, $g - b$ behaves as $\rho_L - \rho_V$, that is, as $\Delta \tilde{T}^{\beta}$.

In a previous publication, a simple power-law expression $B\,|\Delta T^*|^{\beta}$ with free exponent β was fitted in the range 350–374 °C to the quantity $(g-b)/(g+b)$, after it was corrected for the slope of the coexistence-curve diameter.[6] We obtained $T_c = (373.85 \pm 0.01)$ °C (IPTS 1948), the error indicating a doubling of χ^2. The highest point 374 °C, was inconsistent with the other data and, consequently, excluded from the fit. The equations in Appendix A, however, can be used to represent the latent-heat data in a theoretically more satisfying manner, but in a smaller range. We used a preliminary fit of our potential to the *PVT* data to calculate all terms except the leading one. After subtracting the correction terms from the latent heat data (the largest correction, at the lowest temperature, being 1.5%), we fitted the corrected values of $(g-b)/(g+b)$ in the range 370–374 °C again to the asymptotic power law $B\,|\Delta T^*|^{\beta}$ with free exponent β. Again, the highest point seemed inconsistent and was excluded. The results for the dependence of the chi-square of the fit on the choice of T_c are given in Table 2. The optimum value of T_c was 373.84 °C (IPTS 1948), within 0.01° from the one published before,[6] while the value 0.324 ± 0.002 obtained for the exponent β is in excellent agreement with the theory.[7] A larger change, of 0.07 °C, is now required for doubling χ^2, which reflects increased uncertainty because of the smallness of the range. Thus, we have assumed for T_c in this paper

$$T_c = 373.84\ ^\circ\mathrm{C}\ (\mathrm{IPTS}\ 1948) = 647.067\ \mathrm{K}\ (\mathrm{IPTS}\ 1968), \tag{14}$$

given a difference of 0.077 K between readings on IPTS 1948 and IPTS 1968 near the critical point of steam.[18] The corresponding value of P_c is unequivocally defined by the vapor pressure and *PVT* data. We obtained

$$P_c = 22.046\ \mathrm{MPa} \tag{15}$$

with an intrinsic uncertainty of 0.003 MPa and one propagated from the temperature uncertainty δT_c, so that $\delta P_c = (0.267\delta T_c \pm 0.003)$MPa. The value of ρ_c can be inferred from a scaled fit to *PVT* data or from further analysis of the latent heat data.

Table 2. Analysis of the latent heat of steam [22] with varying T_c

T_c, °C (IPTS 1948)	Fit to uncorrected data in the range 330-373.5°C [6]		Fit to corrected data in the range 370-373.5°C [this work]	
	χ^2	β	χ^2	β
373.82			1.98	0.320
373.83	2.19	0.3357	1.84	0.322
373.84	1.06	0.3359	1.84	0.324
373.85	0.55	0.3361	1.96	0.326
373.86	0.66	0.3363	2.19	0.327
373.87	1.35	0.3365	2.53	0.329

Table 3. Critical parameters of steam

	Used here	Estimated uncertainty	Direct determination [31]	Direct determination [33]
P_c, MPa	22.0460	$(0.267\delta T_c \pm 0.003)$	22.045 ± 0.003	22.071 ± 0.002
T_c, K	647.067	±0.1	647.14 ± 0.03	647.19 ± 0.03
ρ_c, kg/m^3	322.778	±2.2		

Our fit to the *PVT* data yields

$$\rho_c = 322.8 \text{ kg/m}^3, \tag{16}$$

while a latent-heat analysis gives $\rho_c = (321.2 \pm 3.1)$ kg/m^3. The values we have obtained for ρ_c from scaled fits of a variety of complexity give optimum ρ_c values no more than 2.2 kg/m^3 removed from the one given. Table 3 summarizes our results for the critical parameters of steam.

The value of T_c determined indirectly is substantially, 0.12 K, below that of the most recent direct determination.[32,33] Against our choice weighs the fact that we have rejected the 374 °C latent heat values, but Osborne himself mentions considerable experimental difficulties at this point.[22] In favor of our choice weighs the extraordinary consistency of latent heat, vapor pressure, and *PVT* data that is obtained as a result (cf. Sec. 7). We hesitate, however, to assign to our current knowledge of T_c an absolute accuracy of better than 0.1 °C.

6. Equation of State

The scaling function parameters a, k_0, k_1, c, b^2, and the pressure background parameters $\tilde{P}_1$, $\tilde{P}_2$, $\tilde{P}_3$, together with ρ_c, were determined by fitting the equation of state to the *PVT* data of Rivkin and co-workers.[23,24] The parameter $\tilde{P}_{11}$ was derived from the known slope of the coexistence curve diameter. The experimental data were assigned absolute weights by propagation of error on the basis of the following error estimates[23,24] and private communication (S. L. Rivkin, 1977):

$$\sigma_P = 0.001 \text{ MPa}, \quad \sigma_T = 0.02 \text{ K}, \quad \sigma_\rho = 0.05\%. \tag{17}$$

The equation fits the experimental *PVT* data with statistic chi-square of 0.58 in the range bounded by

$$200 \text{ kg/m}^3 \leqslant \rho \leqslant 420 \text{ kg/m}^3,$$
$$644 \text{ K} \leqslant T \leqslant 693 \text{ K}. \tag{18}$$

A comparison between the experimental *PVT* data and the values calculated from our equation is presented in Figs. 3–5, in the form of pressure deviations as function of density along various isotherms. The equation reproduces the data within the estimated experimental errors. Some systematic

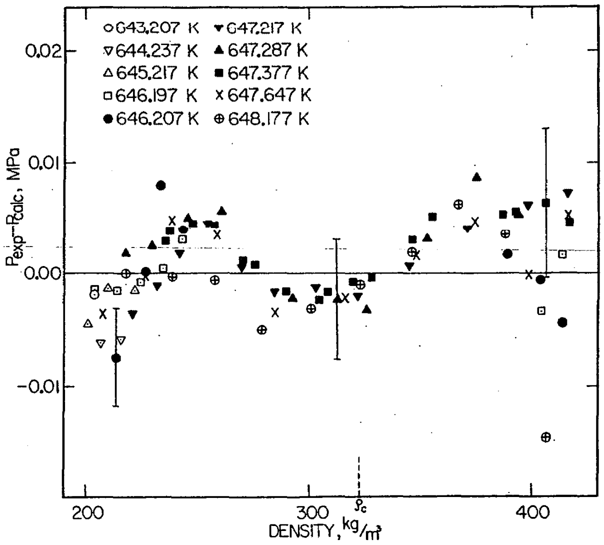

FIG. 3. Deviations of the experimental pressure data of Rivkin *et al.* from the calculated values in the temperature range 643.207 K$\leqslant T \leqslant$648.177 K.

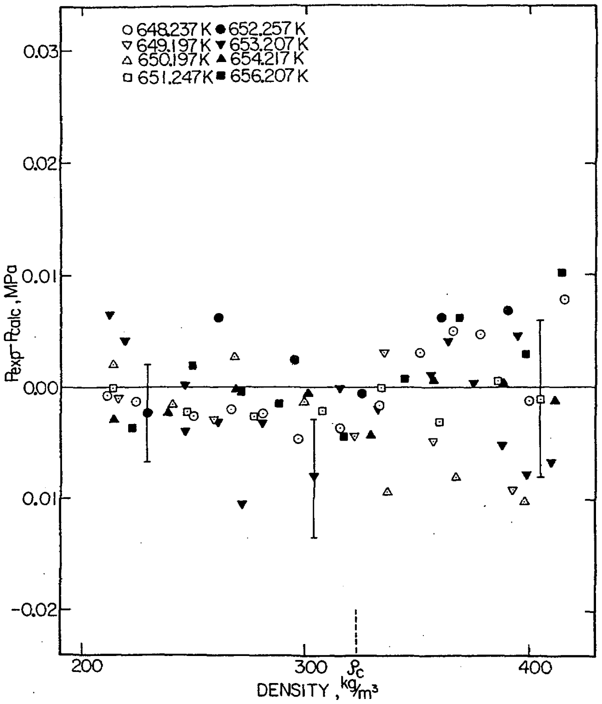

FIG. 4. Deviations of the experimental pressure data of Rivkin *et al.* from the calculated values in the temperature range 648.237 K$\leqslant T \leqslant$ 656.207 K.

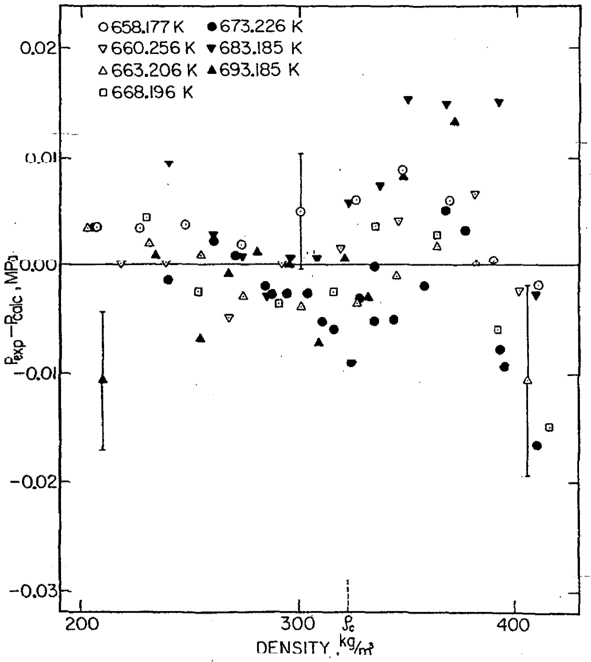

FIG. 5. Deviations of the experimental pressure data of Rivkin *et al.* from the calculated values in the temperature range 658.177 K$\leqslant T$ $\leqslant$693.185 K.

trends, albeit within the estimated error, are visible in the data within a few degrees from the critical point.

7. Vapor Pressure and Latent Heat

The equation of state fitted to the *PVT* data of Rivkin *et al.* can be used to predict the vapor pressure. A comparison with the vapor pressures of Osborne *et al.*[21] is presented in Fig. 6. Although a small systematic trend is present, the data agree with the predicted curve to better than 0.002 MPa. This is very good agreement in view of the fact that the spread by Osborne *et al.* of repeat determinations of the vapor pressure is about ± 0.001 MPa, and the uncertainty of the pressures of Rivkin *et al.* is given as ± 0.001 MPa, with no allowance made for a possible error in temperature.

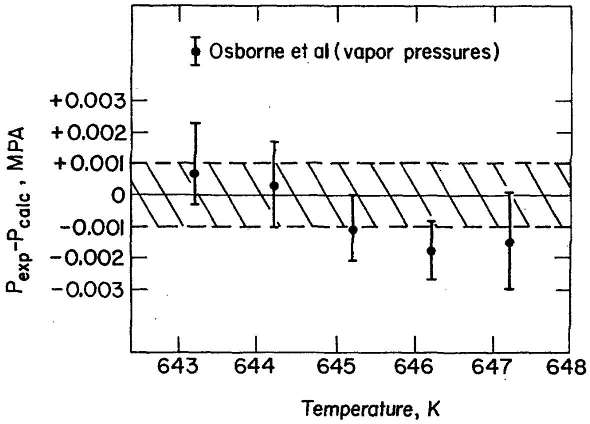

FIG. 6. Comparison between the vapor pressure data of Osborne *et al.* (Ref. 21) and the vapor pressures calculated from the equation of state deduced from the *PVT* data of Rivkin *et al.* (Refs. 23 and 24). The error bars denote the spread of the repeated measurements of (Ref. 21) and the error band corresponds to the direct pressure error of Rivkin's measurements, ± 0.001 MPa.

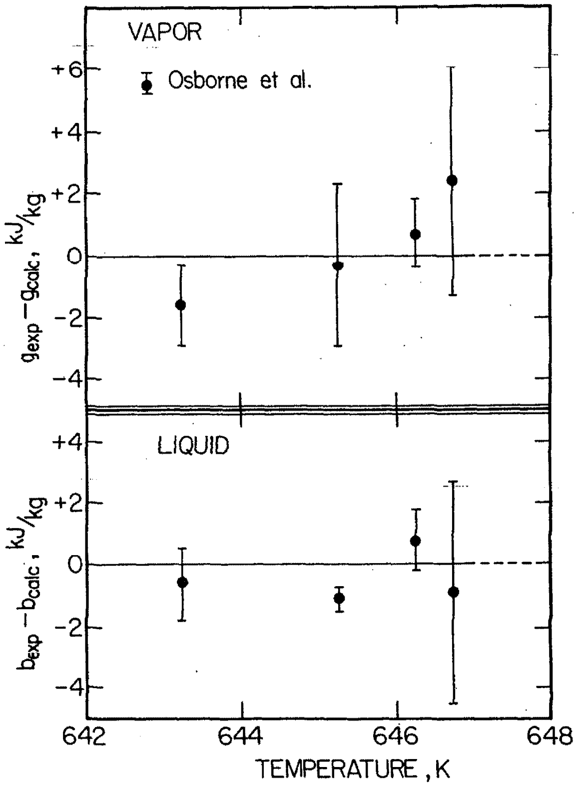

FIG. 7. Comparison between the latent heat data of Osborne *et al.* (Ref. 22) and the values calculated from our surface. The error bars indicate the standard deviations of the repeatedly measured data.

Table 4. Coexistence properties according to our equation, and the data of Osborne et al. [21,22]

T, °C (IPTS 1948)	T, K (IPTS 1968)	P_{vap}(exp) MPa	P_{vap}(calc) MPa	g(exp) kJ/kg	g(calc) kJ/kg	b(exp) kJ/kg	b(calc) kJ/kg
370	643.227	21.0524	21.0517	800.44	802.07	359.35	359.94
371	644.227	21.3056	21.3053		771.25	374.29	376.11
372	645.227	21.5615	21.5625	733.65	733.93	395.48	396.57
373	646.227	21.8217	21.8235	683.05	682.34	427.56	426.72
373.5	646.727		21.9555	642.98	640.54	452.35	453.23
(374.0	647.227	22.0873	22.0888)	(586.62)		(504.3)	

The measured latent heats g, b,[22] can also be compared with our surface. The comparison is presented in Fig. 7. The highest point has been excluded, as discussed in Sec. 5. The experimental data agree to within the spread of repeat measurements with the prediction. The worst departure is 2 kJ/kg. The experimental and predicted values of vapor pressure and latent heats are shown in Table 4.

8. Sources of Error and Measures of Experimental Discrepancies

In the comparison of our surface with data other than those of Rivkin and Osborne, small discrepancies will usually be found that could be due to a number of factors that are usually not known. A major source of error will be the uncertainty in temperature. The critical temperature of steam is quite high, and it is hard to reproduce the temperature scale and eliminate the temperature gradients to better than a few hundredths of a degree unless special metrological precautions are taken. A similar problem arises with the pressure scale which, at 20 MPa, cannot be absolutely reproduced to much better than 1 part in 10^4. Near the critical point, because of the near linearity of the isochores, 1 part in 10^4 in pressure corresponds with approximately 0.01 K, and a discrepancy found between data sets can be quantified either as a difference in temperature scale or in pressure scale, even though the actual cause of the discrepancy may be a different one. For reasons of convenience, we express discrepancies in terms of a difference in temperature scale from that defined by the Osborne–Rivkin data, which we consider as correct in view of their consistency and of the traceability of Osborne's scales.

Because of the strong divergences of the compressibility and the expansion coefficient near a critical point, uncertainties in pressures and temperature, and sample variability due to differences in impurity content, all cause greatly-enlarged errors in the density. Consequently, in those experiments where a property is reported as a function of pressure rather than density, the *location* of the maximum in a diverging quantity such as the isobaric C_p, or in the minimum of a quantity like the isothermal speed of sound, will appear *shifted in density* with respect to the predicted curve. As stated earlier, we will quantify the shift by stating the change in temperature scale required, with respect to that of the Osborne–Rivkin data, in order to eliminate the offset in the location of the extremum.

Quantification of a discrepancy between predicted and measured property data by means of an adjustment of the temperature scale is to be carefully distinguished from the uncertainty in the assignment of a value of T_c to the Rivkin *PVT* data (Sec. 5). The *PVT* data themselves are not sensitive to the choice of T_c. Derived properties, however, are, for instance, the main effect of a change in choice of T_c for the Rivkin data is a change in the predicted value of the peak *height* of a divergent quantity such as C_v while the *location* of the peak is not affected. An offset in peak height will be quantified by giving the amount by which the T_c assignment to Rivkin's data is to be changed in order to eliminate the offset. In general, we will find that the shifts required to eliminate discrepancies are within reasonable expectations of experimental uncertainty. We have found no convincing reasons to doubt the accuracy of the model.

Finally, a source of error in near critical data that is not to be overlooked is that of gravity-induced density gradients in regions where the compressibility is high. As a consequence, the average bulk density in a *PVT* experiment departs from the density at the level at which the pressure is measured, while the theoretical infinity in C_v is suppressed. Moldover *et al.*[34] estimated the range from T_c within which gravity produces errors larger than 1% in the property in question on the critical isochore. These estimates were made for xenon in a cell 1 cm high. In steam, in a 10 cm high cell, these ranges would be 0.15 K for more than 1% error in C_v, and 0.3 K for more than 1% error in density. In the Rivkin *PVT* data, however, the intrinsic pressure error of 0.001 MPa is five times larger than the hydrostatic head in the cell, and therefore gives the dominant uncertainty in the density near the critical point. The C_v data of Baehr *et al.* were not taken within a range of 0.15 K from the critical temperature. Of the C_v data points of Amirchanov *et al.*, the one or two closest to the peak value occasionally are within the gravity-affected range.

9. Caloric Background, Energy, and Speed of Sound

The constants $\tilde{\mu}_i$ in the fundamental equation have to be determined from caloric properties. Of these constants, $\tilde{\mu}_c$ and $\tilde{\mu}_1$ are related to the zeropoints of entropy and energy; they enter into the first derivatives U, H, and S, but not into the second derivatives, specific heats, and speed of sound.

In previous work,[19] the coefficients, $\tilde{\mu}_1$ and $\tilde{\mu}_2$ were determined by fitting the energy equation to the internal energy data of Baehr and co-workers[28] while the coefficient $\tilde{\mu}_3$ was set equal to zero. Recently, however, Erokhin and Kalyanov have reported sound velocity data for steam in an extended region around the critical point.[27] The speed-of-sound data are more sensitive to the choice of the $\tilde{\mu}_i$ $(i > 1)$ so that these constants can be determined more accurately. The speed-of-sound data also indicate that the coefficient $\tilde{\mu}_3$ in the expansion (A6a) for $\tilde{\mu}_0(T)$ cannot be neglected in the temperature and density range under consideration.

If we determine the coefficients $\tilde{\mu}_1$, $\tilde{\mu}_2$, and $\tilde{\mu}_3$ from the energy data of Baehr and co-workers[28,29] in the way described before,[19] we obtain

$$\tilde{\mu}_1 = -22.657 \pm 0.007,$$
$$\tilde{\mu}_2 = -17.750 \pm 0.345,$$
$$\tilde{\mu}_3 = -5.455 \pm 0.407. \tag{19}$$

It should be noted that Baehr's energy values are anchored to those of the surface of Keenan *et al.* at 350 °C. On the other hand, if we determine $\tilde{\mu}_2$ and $\tilde{\mu}_3$ from the speed-of-sound data of Erokhin and Kalyanov[27] we obtain

$$\tilde{\mu}_2 = -17.888 \pm 0.010, \tilde{\mu}_3 = -4.933 \pm 0.061. \tag{20}$$

The two sets of values are in excellent agreement, but the values deduced for $\tilde{\mu}_2$ and $\tilde{\mu}_3$ from speed-of-sound data evidently have a higher accuracy. These values, Eq. (20), have been adopted in this work.

This leaves the zeropoint constants $\tilde{\mu}_c$ and $\tilde{\mu}_1$ to be determined. This was done by identifying the energy and entropy of our surface at $T = 648.15$ K, $\rho = 230$ kg/m^3, with those of the global surface proposed by Haar, Gallagher, and Kell.[35] The results are

$$\tilde{\mu}_c = -11.233, \tilde{\mu}_1 = -22.655. \tag{21}$$

The value for $\tilde{\mu}_1$ is in excellent agreement with that[19] obtained from a direct fit to the energy data of Baehr *et al.*[28,29]

After all constants in the equation have thus been determined (Appendix B), comparisons will be made with experimental data for speed-of-sound, energy, and specific heat at constant volume.

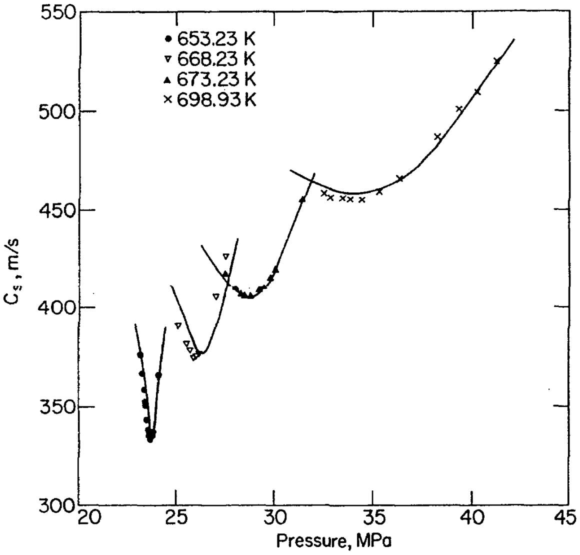

FIG. 9. The speed of sound as a function of pressure in the temperature range 653.23 K$\leqslant T \leqslant$698.93 K. The data are those of Erokhin and Kalyanov (Ref. 27). The curves represent values predicted by our equation.

A comparison of the speed-of-sound data of Erokhin and Kalyanov and the values calculated from our equation is presented in Figs. 8 and 9. The equation reproduces correctly the magnitude of the speed-of-sound minima at all temperatures indicating agreement with the value of T_c ascribed to Rivkin's data (cf. Sec. 8). In Fig. 10 we show how much the fit to the speed-of-sound data closest to the critical point improves by lowering their temperature values by 0.06 K; the location of the minima improves while the good agreement of the minimum values is retained.

In our opinion, a combined uncertainty in realization of

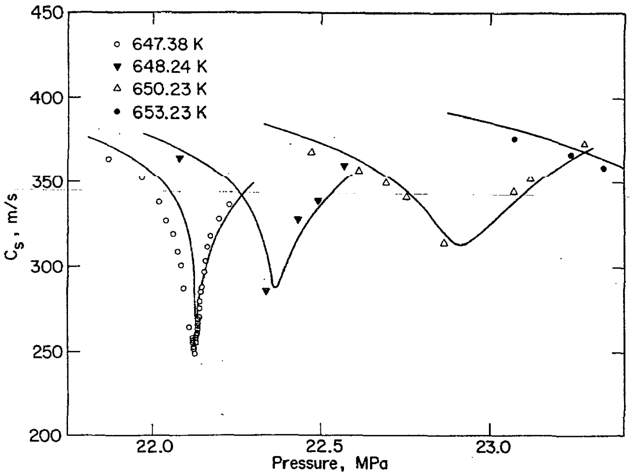

FIG. 8. The speed of sound as a function of pressure in the temperature range 647.38 K$\leqslant T \leqslant$653.23 K. The data points are those of Erokhin and Kalyanov (Ref. 27). The curves represent values predicted by our equation.

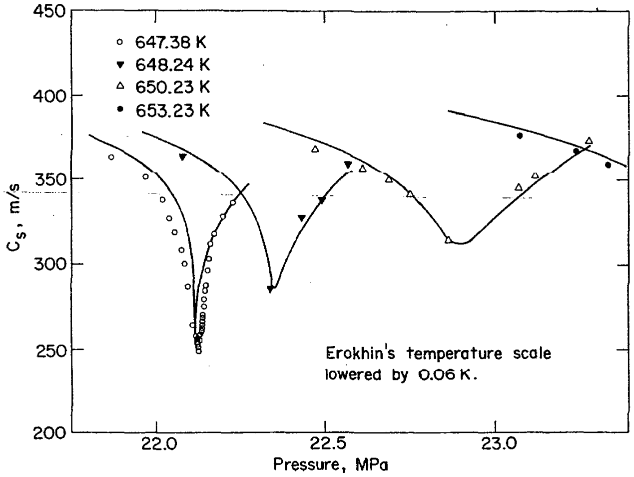

FIG. 10. Another comparison of the speed of sound in the same range as that of Fig. 8, but after subtracting 0.06 K from the temperature values of (Ref. 27). This temperature adjustment eliminates most of the systematic difference between data and prediction.

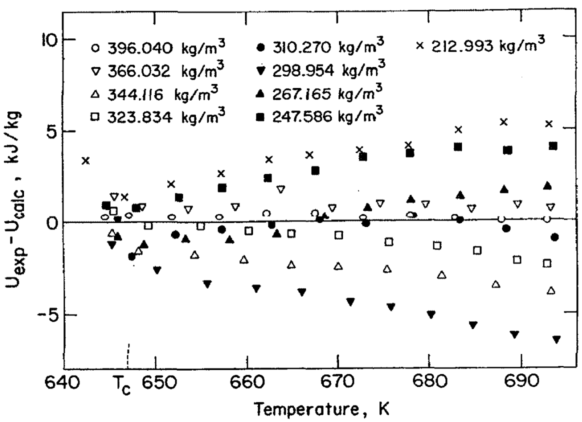

FIG. 11. Deviations of the experimental energy data of Baehr *et al.* (Ref. 28) from the calculated values.

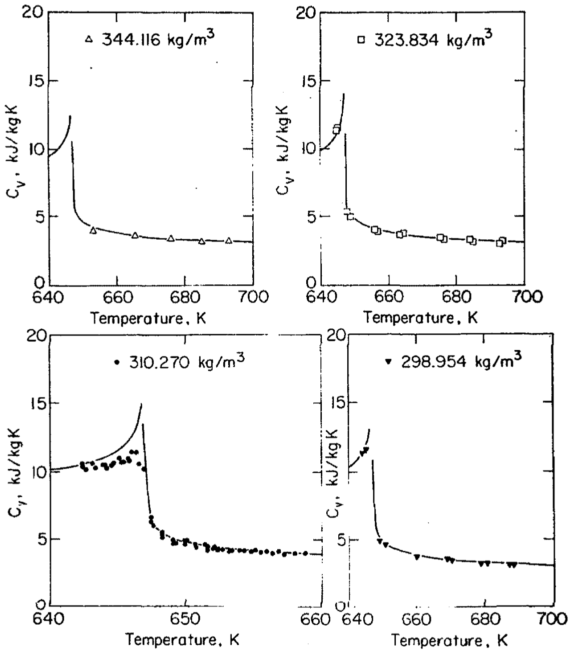

FIG. 12. The specific heat $\overline{C}_v = \Delta U/\Delta T$ as a function of temperature. The symbols represent the data reported by Baehr and Schomäcker (Ref. 29). The curves represent values predicted by our equation.

pressure and temperature measurement and in sample composition, as expressed by 0.06 K in temperature, is not unreasonable.

The comparison with the energy data of Baehr *et al.*[28] is shown in Fig. 11. Although the deviations along each isochore have a systematic character, the departures from the surface are well within the claimed accuracy of the data. In Fig. 12 we compare with the C_v data reported by Baehr and Schomäcker.[29] The data were obtained as the ratio of energy increments ΔU over temperature increments ΔT, with ΔT of *the order of degrees; the predicted curves in Fig. 12 represent* the quantity $\Delta U/\Delta T$, with ΔT the experimental temperature increments. In general, the data are accurately consistent with the weakly diverging specific heat C_v predicted by the scaling laws. An exception is formed by some of the two-phase data on the 310.27 kg/m^3 isochore that do not rise steeply enough. No adjustment of the temperature scale was required.

Amirkhanov, Kerimov, and co workers have, at various occasions, published detailed measurements of the specific heat C_v near the phase boundary. A previous analysis indicated that not all these data appeared to be mutually consistent.[6]

In a book published since by Amirkhanov *et al.*,[30] the older measurements are scrutinized; several sets being discarded because of the presence of air. This book, however, *also contains new C_v data not hitherto published.* Those are the ones we compare with here. A problem we encountered is that the jump in C_v, from a 2-phase to a 1-phase value, occurs between 374.10 and 374.18 °C (IPTS 1948), which is 0.31 °C above our choice of T_c. After all of Amirkhanov's temperatures were lowered by this amount, (a much larger adjustment than for any of the other data, and one for which we have no satisfactory explanation), our surface gives a very close representation of this anomalous region of C_v (Fig. 13). Note especially how well the steep rise in C_v is represented on the 315.457 kg/m^3 isochore where the two-phase C_v doubles in a span of a few K. As mentioned, Baehr's nearby isochore did not show this predicted rise in the two-phase region (Fig. 12).

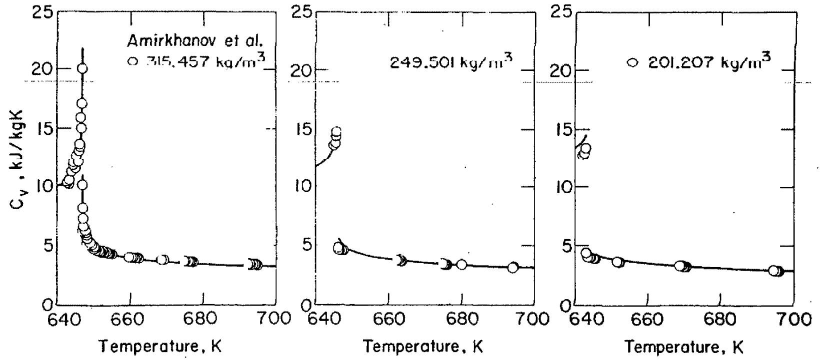

FIG. 13. The specific heat C_v as a function of temperature. The circles represent the data of Amirkhanov *et al.* (Ref. 30) after 0.31 K was subtracted from all experimental temperatures.

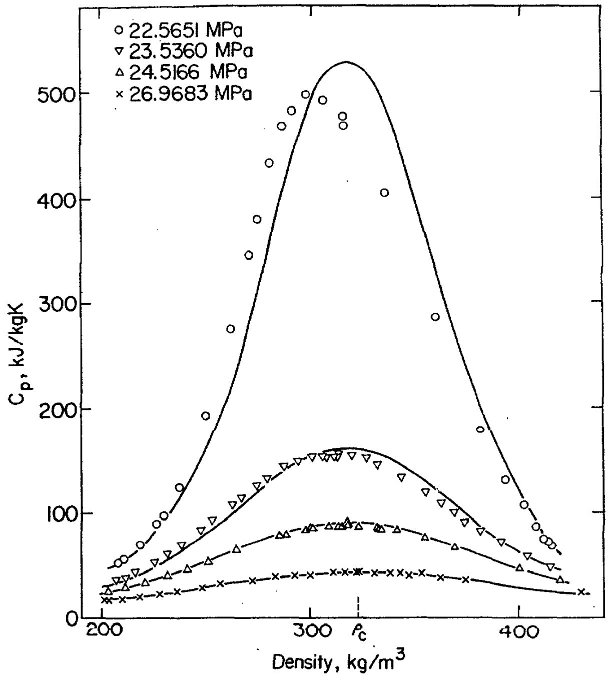

FIG. 14. The data of Sirota and Maltsev (Ref. 26) for the specific heat C_p along isobars, and as a function of density, are compared with the curves generated by our surface. The peak heights are well predicted but there is a small shift in density between the experimental and predicted values.

10. Specific Heat C_p

A comparison between the values calculated for the specific heat C_p and the experimental data of Sirota and Maltsev[26] is presented in Fig. 14. The data were taken as enthalpy increments in given finite temperature intervals and the points shown in the plot represent values of $\Delta H/\Delta T$. Our predicted values are, likewise, values of $\Delta H/\Delta T$ over the experimental temperature intervals. Since the temperature intervals ΔT were small and smoothly varying, and the points closely spaced, smooth curves could be drawn through the predicted points. The equation reproduces the experimental maxima in C_p at all temperatures; the maximum difference in the peak values is about 5% to 6% at the 22.5651 MPa isobar.

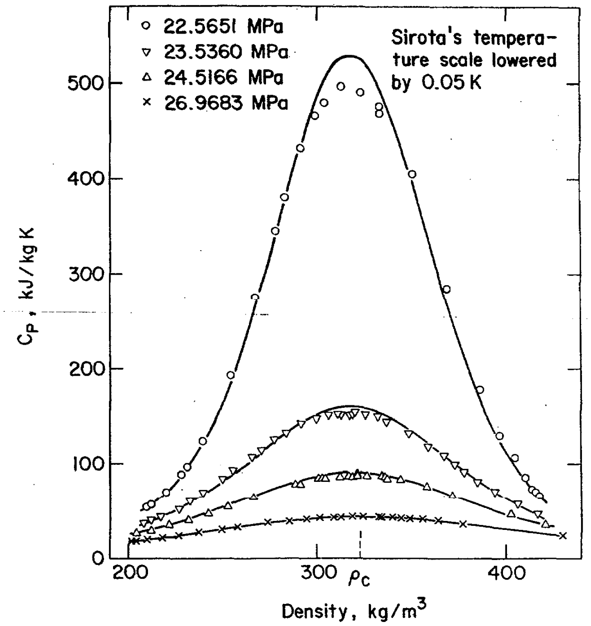

FIG. 15. A lowering of the experimental temperatures by 0.05 K eliminates the offset between C_p experiment and prediction observed in Fig. 14.

This difference of peak heights could be further reduced by ascribing a lower value of T_c to the *PVT* data, as we did before[19]; cf. also Sec. 8. Values for T_c substantially lower than the one used here, however, although reconcilable with the *PVT* data, are less compatible with Osborne's latent heats. In Fig. 14 the comparison is made with the C_p data plotted as function of density, rather than temperature. The scaled equation was used to transform the experimental pressures to densities. The plot reveals a small offset in peak location between the experimental data and the predicted curve, as noted earlier.[19] The effect can be eliminated by lowering Sirota's temperature values by 0.05 K, as demonstrated in Fig. 15. We conclude that the C_p data or Sirota and Maltsev and the *PVT* data of Rivkin *et al.* are consistent at an acceptable level.

11. Critical Amplitude Ratios

The scaling function parameter b^2, although predicted to be universal, was actually determined by us from the fit. Universality of b^2 implies universality of the critical amplitude ratios [see Eqs. (A38)–(A41) in Appendix A]. The amplitude ratios we obtain in our fit are indeed in agreement with the values predicted theoretically, as shown in Table 5.

Table 5. Critical amplitude ratios

	Theory [20]	Theory [20]	Our results
A^+/A^-	0.51	0.55	0.532
Γ^+/Γ^-	5.07	4.80	4.886
$\Gamma^+ DB^{\delta-1}$	1.75	1.6	1.693
$A^+\Gamma^+/B^2$	0.059	0.066	0.0568
	series expansion Ising	ϵ-expansion renormalization group	

12. Thermodynamic Tables

Tables of the thermodynamic properties pressure, enthalpy, entropy, energy, specific heats, and speed of sound have been prepared along closely spaced isochores, and as a function of temperature.

In addition, a table of saturation properties has been constructed. The tables are given in Appendix C.

13. Discussion

We have presented a description of the thermodynamic behavior of steam in the range 200–420 kg/m^3, 643 693 K, including the critical point. The fundamental equation is of the form predicted by the modern theory of critical phenomena, and includes a correction-to-scaling term and a device for introducing gas–liquid asymmetry. In total, 16 adjustable parameters were used.

The *PVT* data of Rivkin *et al.* were found to be consistent with the vapor pressure and latent-heat data of Osborne *et al.* to a high degree of accuracy.

The speed-of-sound data of Erokhin and Kalyanov and C_p data of Sirota and Maltsev are consistent with the *PVT* of Rivkin after small adjustments of the respective temperature scales, of the order of a few hundredths of a degree. The energy and C_v data of Baehr *et al.* are also consistent with the *PVT* data, but the tolerances of these data are somewhat larger than those of C_p and speed of sound.

The C_v data of Amirkhanov *et al.* which include values very near the critical point, are represented in fine detail by our surface. A large adjustment of the temperature scale, -0.31 K, however, was needed.

There exists an internationally recognized formulation for the thermodynamic surface of steam recommended for scientific and industrial use.[36] We do not present a comparison here with the predictions from this international formulation or other global formulations. The reason is that the International Association for the Properties of Steam is in the process of adopting a new formulation which will be more accurate and valid in a much larger range.[35,37] In this paper the quality of our fundamental equation for the critical region has been judged solely on the basis of a comparison with the experimental data.

Acknowledgments

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Appendix A. Revised and Extended Scaling Equations for the Thermodynamic Properties of Fluids

A 1. Reduced thermodynamic quantities

$$\tilde{T} = -\frac{T_c}{T}, \quad \tilde{\mu} = \frac{\mu}{T}\cdot\frac{\rho_c T_c}{P_c}, \quad \tilde{P} = \frac{P}{T}\cdot\frac{T_c}{P_c},$$

$$\tilde{\rho} = \frac{\rho}{\rho_c}, \quad \tilde{U} = \frac{U}{V}\cdot\frac{1}{P_c}, \quad \tilde{S} = \frac{S}{V}\cdot\frac{T_c}{P_c}, \tag{A1}$$

$$\tilde{A} = \frac{A}{VT}\cdot\frac{T_c}{P_c}, \quad \tilde{H} = \frac{H}{VT}\cdot\frac{T_c}{P_c}, \quad \tilde{\chi}_T = \left(\frac{\partial\tilde{\rho}}{\partial\tilde{\mu}}\right)_T,$$

$$\tilde{C}_v = \frac{C_v}{V}\frac{T_c}{P_c}, \quad \tilde{C}_P = \frac{C_P}{V}\cdot\frac{T_c}{P_c}.$$

(T is temperature, μ is chemical potential, P is pressure, ρ is density, U is energy, S is entropy, A is helmholtz free energy, H is enthalpy, V is volume, C_v is heat capacity at constant V, C_p is heat capacity at constant P).

A 2. Thermodynamic relations

$$\begin{aligned} \mathrm{d}\widetilde{P} &= \widetilde{U}\,\mathrm{d}\widetilde{T} + \tilde{\rho}\,\mathrm{d}\tilde{\mu}, \\ \mathrm{d}\widetilde{A} &= -\widetilde{U}\,\mathrm{d}\widetilde{T} + \tilde{\mu}\,\mathrm{d}\tilde{\rho}, \\ \mathrm{d}\widetilde{H} &= -\widetilde{T}\,\mathrm{d}\widetilde{U} + \tilde{\rho}\,\mathrm{d}\tilde{\mu}, \\ \mathrm{d}\widetilde{S} &= -\widetilde{T}\,\mathrm{d}\widetilde{U} - \tilde{\mu}\,\mathrm{d}\tilde{\rho}, \end{aligned} \tag{A2}$$

with

$$\begin{aligned} \widetilde{A} &= \tilde{\rho}\tilde{\mu} - \widetilde{P}, \\ \widetilde{H} &= \widetilde{P} - \widetilde{T}\widetilde{U}, \\ \widetilde{S} &= \widetilde{H} - \tilde{\rho}\tilde{\mu} = -\widetilde{T}\widetilde{U} - \widetilde{A}. \end{aligned} \tag{A3}$$

A 3. Fundamental equations

$$\Delta\widetilde{T} = \widetilde{T} + 1, \tag{A4a}$$

$$\Delta\tilde{\mu} = \tilde{\mu} - \tilde{\mu}_0(\widetilde{T}), \tag{A4b}$$

$$\widetilde{P} = \widetilde{P}_0(\widetilde{T}) + \Delta\tilde{\mu} + \widetilde{P}_{11}\Delta\tilde{\mu}\Delta\widetilde{T} + \Delta\widetilde{P}, \tag{A5}$$

with

$$\tilde{\mu}_0(\widetilde{T}) = \tilde{\mu}_c + \sum_{j=1}^{3}\tilde{\mu}_j(\Delta\widetilde{T})^j, \tag{A6a}$$

$$\widetilde{P}_0(\widetilde{T}) = 1 + \sum_{j=1}^{3}\widetilde{P}_j(\Delta\widetilde{T})^j. \tag{A6b}$$

A 4. Derived thermodynamic quantities

$$\tilde{\rho} = 1 + \widetilde{P}_{11}\Delta\widetilde{T} + \left(\frac{\partial\Delta\widetilde{P}}{\partial\Delta\tilde{\mu}}\right)_{\Delta\widetilde{T}}, \tag{A7}$$

$$\widetilde{U} = \frac{\mathrm{d}\widetilde{P}_0}{\mathrm{d}\widetilde{T}} - \tilde{\rho}\frac{\mathrm{d}\tilde{\mu}_0}{\mathrm{d}\widetilde{T}} + \widetilde{P}_{11}\Delta\mu + \left(\frac{\partial\Delta\widetilde{P}}{\partial\Delta\widetilde{T}}\right)_{\Delta\tilde{\mu}}, \tag{A8}$$

$$\widetilde{\chi}_T = \left(\frac{\partial^2\Delta\widetilde{P}}{\partial\Delta\tilde{\mu}^2}\right)_{\Delta\widetilde{T}}, \tag{A9}$$

$$\begin{aligned} \left(\frac{\partial\widetilde{P}}{\partial\widetilde{T}}\right)_{\tilde{\rho}} &= \frac{\mathrm{d}\widetilde{P}_0}{\mathrm{d}\widetilde{T}} + \widetilde{P}_{11}\left[\Delta\tilde{\mu} - \frac{\tilde{\rho}}{\widetilde{\chi}_T}\right] \\ &\quad + \left(\frac{\partial\Delta\widetilde{P}}{\partial\Delta\widetilde{T}}\right)_{\Delta\tilde{\mu}} - \frac{\tilde{\rho}}{\widetilde{\chi}_T}\frac{\partial^2\Delta\widetilde{P}}{\partial\Delta\tilde{\mu}\partial\Delta\widetilde{T}}, \end{aligned} \tag{A10}$$

$$\begin{aligned} \frac{\widetilde{C}_v}{\widetilde{T}^2} &= \frac{\mathrm{d}^2\widetilde{P}_0}{\mathrm{d}\widetilde{T}^2} - \tilde{\rho}\frac{\mathrm{d}^2\tilde{\mu}_0}{\mathrm{d}\widetilde{T}^2} - \frac{\widetilde{P}_{11}{}^2}{\widetilde{\chi}_T} \\ &\quad + \left(\frac{\partial^2\Delta\widetilde{P}}{\partial\Delta\widetilde{T}^2}\right)_{\Delta\tilde{\mu}} - \frac{2\widetilde{P}_{11}}{\widetilde{\chi}_T}\frac{\partial^2\Delta\widetilde{P}}{\partial\Delta\tilde{\mu}\partial\Delta\widetilde{T}} \\ &\quad - \frac{1}{\widetilde{\chi}_T}\left(\frac{\partial^2\Delta\widetilde{P}}{\partial\Delta\tilde{\mu}\partial\Delta\widetilde{T}}\right)^2, \end{aligned} \tag{A11}$$

$$\widetilde{C}_P = \widetilde{C}_v + \frac{\widetilde{\chi}_T}{\tilde{\rho}^2}\left[\widetilde{P} - \widetilde{T}\left(\frac{\partial\widetilde{P}}{\partial\widetilde{T}}\right)_{\tilde{\rho}}\right]^2. \tag{A12}$$

A 5. Critical exponents

$$\begin{aligned} &\alpha_0 = \alpha, \quad \alpha_1 = \alpha - \Delta_1, \\ &\beta_0 = \beta, \quad \beta_1 = \beta + \Delta_1, \\ &\gamma_0 = \gamma, \quad \gamma_1 = \gamma - \Delta_1, \end{aligned} \tag{A13}$$

with

$$2 - \alpha = \beta(\delta + 1), \quad \gamma = \beta(\delta - 1). \tag{A14}$$

A 6. Parametric equations for singular terms

$$\Delta\tilde{\mu} = r^{\beta\delta}a\theta(1 - \theta^2), \tag{A15}$$

$$\Delta\widetilde{T} = r(1 - b^2\theta^2) - c\Delta\tilde{\mu}, \tag{A16}$$

$$\Delta\widetilde{P} = \sum_{i=0}^{1} r^{2-\alpha_i}ak_ip_i(\theta), \tag{A17}$$

$$\left(\frac{\partial\Delta\widetilde{P}}{\partial\Delta\tilde{\mu}}\right)_{\Delta\widetilde{T}} = \sum_{i=0}^{1}\left[r^{\beta_i}k_i\theta + cr^{1-\alpha_i}ak_is_i(\theta)\right], \tag{A18}$$

$$\left(\frac{\partial\Delta\widetilde{P}}{\partial\Delta\widetilde{T}}\right)_{\Delta\tilde{\mu}} = \sum_{i=0}^{1} r^{1-\alpha_i}ak_is_i(\theta), \tag{A19}$$

$$\begin{aligned} \left(\frac{\partial^2\Delta\widetilde{P}}{\partial\Delta\tilde{\mu}^2}\right)_{\Delta\widetilde{T}} &= \sum_{i=0}^{1}\Big[r^{-\gamma_i}\frac{k_i}{a}u_i(\theta) + 2cr^{\beta_i-1}k_iv_i(\theta) \\ &\quad + c^2r^{-\alpha_i}ak_iw_i(\theta)\Big], \end{aligned} \tag{A20}$$

$$\frac{\partial^2\Delta\widetilde{P}}{\partial\Delta\tilde{\mu}\Delta\widetilde{T}} = \sum_{i=0}^{1}\left[r^{\beta_i-1}k_iv_i(\theta) + cr^{-\alpha_i}ak_iw_i(\theta)\right], \tag{A21}$$

$$\left(\frac{\partial^2\Delta\widetilde{P}}{\partial\Delta\widetilde{T}^2}\right)_{\Delta\tilde{\mu}} = \sum_{i=0}^{1} r^{-\alpha_i}ak_iw_i(\theta). \tag{A22}$$

A 7. Auxiliary functions

$$p_i(\theta) = p_{0i} + p_{2i}\theta^2 + p_{4i}\theta^4, \tag{A23}$$

$$s_i(\theta) = s_{0i} + s_{2i}\theta^2, \quad s_i'(\theta) = 2s_{2i}\theta, \tag{A24}$$

$$q(\theta) = 1 + \{b^2(2\beta\delta - 1) - 3\}\theta^2 - b^2(2\beta\delta - 3)\theta^4, \tag{A25}$$

$$u_i(\theta) = [1 - b^2(1 - 2\beta_i)\theta^2]/q(\theta), \tag{A26}$$

$$v_i(\theta) = [\beta_i(1 - 3\theta^2)\theta - \beta\delta(1 - \theta^2)\theta]/q(\theta), \tag{A27}$$

$$\begin{aligned} w_i(\theta) &= [(1 - \alpha_i)(1 - 3\theta^2)s_i(\theta) \\ &\quad - \beta\delta(1 - \theta^2)\theta s_i'(\theta)]/q(\theta), \end{aligned} \tag{A28}$$

with

$$\begin{aligned} p_{0i} &= +\frac{\beta\delta - 3\beta_i - b^2\alpha_i\gamma_i}{2b^4(2 - \alpha_i)(1 - \alpha_i)\alpha_i}, \\ p_{2i} &= -\frac{\beta\delta - 3\beta_i - b^2\alpha_i(2\beta\delta - 1)}{2b^2(1 - \alpha_i)\alpha_i}, \\ p_{4i} &= +\frac{2\beta\delta - 3}{2\alpha_i}, \end{aligned} \tag{A29}$$

$$\begin{aligned} s_{0i} &= (2 - \alpha_i)p_{0i}, \\ s_{2i} &= -\frac{\beta\delta - 3\beta_i}{2b^2\alpha_i}. \end{aligned} \tag{A30}$$

A 8. Two-phase properties

Variables:

$$\begin{aligned} &\theta = \pm 1, \\ &\Delta\tilde{\mu} = 0, \\ &\Delta\widetilde{T} = r(1 - b^2). \end{aligned} \tag{A31}$$

Vapor pressure:

$$\widetilde{P}_{\mathrm{vap}} = \widetilde{P}_0(\widetilde{T}) + \sum_{i=0}^{1} r^{2-\alpha_i}ak_ip_i(1), \tag{A32}$$

Coexisting densities:

$$\frac{\tilde{\rho}_L + \tilde{\rho}_V}{2} = 1 + \widetilde{P}_{11}\Delta\widetilde{T} + \sum_{i=0}^{1} c r^{1-\alpha_i} a k_i s_i(1),$$

$$\frac{\tilde{\rho}_L - \tilde{\rho}_V}{2} = \sum_{i=0}^{1} r^{\beta_i} k_i. \qquad \text{(A33)}$$

Helmholtz free energy:

$$\widetilde{A} = \tilde{\rho}\tilde{\mu}_0(\widetilde{T}) - \widetilde{P}_0(T) - \sum_{i=0}^{1} r^{2-\alpha_i} a k_i p_i(1). \qquad \text{(A34)}$$

Energy:

$$\widetilde{U} = \frac{\mathrm{d}\widetilde{P}_0}{\mathrm{d}\widetilde{T}} - \tilde{\rho}\,\frac{\mathrm{d}\tilde{\mu}_0(T)}{\mathrm{d}\widetilde{T}} + \frac{1}{1-b^2} \times \sum_{i=0}^{1} (2-\alpha_i) r^{1-\alpha_i} a k_i p_i(1). \qquad \text{(A35)}$$

Entropy:

$$\widetilde{S} = -\tilde{\rho}\tilde{\mu}_0(\widetilde{T}) + \widetilde{P}_0(\widetilde{T}) - \widetilde{T}\left[\frac{\mathrm{d}\widetilde{P}_0}{\mathrm{d}\widetilde{T}} - \rho\,\frac{\mathrm{d}\tilde{\mu}_0}{\mathrm{d}\widetilde{T}}\right] + \sum_{i=0}^{1} r^{2-\alpha_i} a k_i p_i(1) - \frac{\widetilde{T}}{1-h^2}\sum_{i=0}^{1}(2-\alpha_i) r^{1-\alpha_i} a k_i p_i(1). \qquad \text{(A36)}$$

Specific heat C_v:

$$\frac{\widetilde{C}_v}{\widetilde{T}^2} = \frac{\mathrm{d}^2\widetilde{P}_0}{\mathrm{d}\widetilde{T}^2} - \tilde{\rho}\,\frac{\mathrm{d}^2\tilde{\mu}_0}{\mathrm{d}\widetilde{T}^2} + \left(\frac{1}{1-b^2}\right)^2 \sum_{i=0}^{1} (2-\alpha_i)(1-\alpha_i) r^{-\alpha_i} a k_i p_i(1). \qquad \text{(A37)}$$

A 9. Critical amplitude ratios

$$\frac{A^+}{A^-} = \frac{(b^2-1)^{2-\alpha} p_{00}}{p_{00} + p_{20} + p_{40}}, \qquad \text{(A38)}$$

$$\frac{\Gamma^+}{\Gamma^-} = \frac{2}{(b^2-1)^{\gamma-1}\{1 - b^2(1-2\beta)\}}, \qquad \text{(A39)}$$

$$\Gamma^+ D B^{\delta-1} = \frac{b^{\delta-3}}{(b^2-1)^{\gamma-1}}, \qquad \text{(A40)}$$

$$\frac{A^+\Gamma^+}{B^2} = (2-\alpha)(1-\alpha)\alpha(b^2-1)^{2\beta} p_{00}. \qquad \text{(A41)}$$

Appendix B. Parameter Values for the Thermodynamic Surface of Steam in the Critical Region

TABLE B1. Parameter values[a]

Critical exponents:	$\beta = 0.325$[b]
	$\delta = 4.82$[b]
	$\Delta_1 = 0.50$[b]
Critical parameters:	$T_c = 647.067$ K[c]
	$\rho_c = 322.778$ kg/m^3 [d]
	$P_c = 22.0460$ MPa[e]
Parameters in scaling functions:	$a = +23.667$[d]
	$k_0 = +1.4403$[d]
	$k_1 = +0.2942$[d]
	$c = -0.01776$[d]
	$b^2 = +1.3757$[d]
Pressure background parameters:	$\widetilde{P}_1 = +6.8445$[d]
	$\widetilde{P}_2 = -25.4915$[d]
	$\widetilde{P}_3 = +5.238$[d]
	$\widetilde{P}_{11} = +0.4918$[f]
Caloric background parameters:	$\tilde{\mu}_c = -11.233$[g]
	$\tilde{\mu}_1 = -22.655$[g]
	$\tilde{\mu}_2 = -17.888$[h]
	$\tilde{\mu}_3 = -4.933$[h]

[a] The number of decimals given exceeds the number of significant decimals so as to retain precision in complex calculations.
[b] Fixed from theory.
[c] From latent heat data of Osborne *et al.*
[d] From fit to *PVT* data of Rivkin *et al.*
[e] Derived from vapor pressure curve.
[f] From known coexistence curve diameter.
[g] From identification of our surface with that of Haar *et al.* at $T = 648.15$ K and $\rho = 230$ kg/m^3.
[h] From fit to speed of sound data of Erokhin and Kalyanov.

Appendix C. Tables of Thermodynamic Properties of Steam in the Critical Region

The uncertainty of the tabulated values is generally a few units in the last decimal given. However, in special cases, such as for the densities near ρ_c and T_c and the specific heats in the region where they vary rapidly, the uncertainty will be much larger.

Table C1. Thermodynamic properties along isochores at regular temperature increments.

Temp.	Pressure	Density	Internal Energy	Enthalpy	Entropy	C_p	C_v	Velocity of Sound	Phase Region
K	MPa	kg/m^3	kJ/kg	kJ/kg	kJ/kg.K	kJ/kg.K		m/s	
643.0	20.995	200.0	2234.5	2339.5	4.8090		14.46		2
644.0	21.184	200.0	2239.3	2345.2	4.8165	81.4	4.50	364.9	1
645.0	21.371	200.0	2243.8	2350.6	4.8234	71.0	4.39	368.5	1
646.0	21.557	200.0	2248.1	2355.9	4.8301	63.1	4.29	371.8	1
647.0	21.741	200.0	2252.4	2361.1	4.8367	57.0	4.21	374.8	1
648.0	21.925	200.0	2256.5	2366.2	4.8431	52.1	4.14	377.7	1
650.0	22.290	200.0	2264.7	2376.1	4.8557	44.6	4.02	383.0	1
655.0	23.191	200.0	2284.2	2400.2	4.8856	33.2	3.80	394.9	1
660.0	24.081	200.0	2302.8	2423.2	4.9139	26.7	3.64	405.4	1
665.0	24.962	200.0	2320.7	2445.5	4.9409	22.6	3.52	415.0	1
670.0	25.836	200.0	2338.1	2467.2	4.9669	19.6	3.43	424.0	1
675.0	26.705	200.0	2355.0	2488.5	4.9921	17.4	3.35	432.6	1
680.0	27.568	200.0	2371.5	2509.4	5.0165	15.8	3.28	440.8	1
685.0	28.427	200.0	2387.8	2529.9	5.0403	14.4	3.22	448.7	1
690.0	29.283	200.0	2403.7	2550.1	5.0635	13.3	3.17	456.4	1
695.0	30.135	200.0	2419.4	2570.1	5.0862	12.4	3.12	463.9	1
643.0	20.995	220.0	2170.7	2266.1	4.6948		13.49		2
644.0	21.247	220.0	2184.4	2280.9	4.7161		13.95		2
645.0	21.490	220.0	2196.0	2293.7	4.7342	155.6	4.88	346.9	1
646.0	21.696	220.0	2200.8	2299.4	4.7416	121.7	4.69	352.3	1
647.0	21.899	220.0	2205.4	2305.0	4.7488	100.9	4.54	356.7	1
648.0	22.102	220.0	2209.9	2310.3	4.7557	86.6	4.42	360.7	1
650.0	22.504	220.0	2218.5	2320.8	4.7690	67.9	4.24	367.6	1
655.0	23.495	220.0	2238.9	2345.7	4.8003	44.9	3.94	381.9	1
660.0	24.474	220.0	2258.1	2369.4	4.8295	33.9	3.75	394.0	1
665.0	25.444	220.0	2276.5	2392.1	4.8572	27.4	3.61	404.9	1
670.0	26.408	220.0	2294.2	2414.3	4.8838	23.2	3.49	414.9	1
675.0	27.366	220.0	2311.5	2435.9	4.9094	20.2	3.40	424.3	1
680.0	28.319	220.0	2328.3	2457.0	4.9342	17.9	3.33	433.3	1
685.0	29.269	220.0	2344.8	2477.8	4.9584	16.2	3.26	441.9	1
690.0	30.215	220.0	2360.9	2498.3	4.9819	14.8	3.21	450.2	1
695.0	31.158	220.0	2376.8	2518.5	5.0048	13.7	3.16	458.3	1
643.0	20.995	240.0	2117.4	2204.9	4.5997		12.69		2
644.0	21.247	240.0	2130.3	2218.8	4.6197		13.10		2
645.0	21.504	240.0	2143.7	2233.3	4.6405		13.73		2
646.0	21.759	240.0	2156.7	2247.3	4.6606	319.4	5.28	328.9	1
647.0	21.981	240.0	2161.8	2253.4	4.6685	208.9	4.95	337.2	1
648.0	22.201	240.0	2166.6	2259.1	4.6759	157.7	4.74	343.3	1
650.0	22.638	240.0	2175.8	2270.1	4.6901	107.1	4.46	352.8	1
655.0	23.714	240.0	2197.0	2295.8	4.7226	60.3	4.06	370.3	1
660.0	24.778	240.0	2216.7	2319.9	4.7525	42.3	3.83	384.2	1
665.0	25.833	240.0	2235.4	2343.0	4.7808	32.8	3.67	396.4	1
670.0	26.884	240.0	2253.4	2365.4	4.8078	26.9	3.54	407.5	1
675.0	27.930	240.0	2270.9	2387.2	4.8337	22.9	3.44	417.8	1
680.0	28.972	240.0	2287.9	2408.6	4.8589	20.1	3.36	427.5	1
685.0	30.012	240.0	2304.5	2429.6	4.8832	17.9	3.29	436.8	1
690.0	31.049	240.0	2320.8	2450.2	4.9069	16.2	3.23	445.8	1
695.0	32.083	240.0	2336.9	2470.5	4.9301	14.8	3.18	454.4	1
643.0	20.995	260.0	2072.4	2153.1	4.5192		12.00		2
644.0	21.247	260.0	2084.6	2166.3	4.5381		12.39		2
645.0	21.504	260.0	2097.2	2179.9	4.5577		12.96		2
646.0	21.764	260.0	2110.7	2194.4	4.5786		14.02		2
647.0	22.016	260.0	2121.3	2206.0	4.5950	554.6	5.52	314.4	1
648.0	22.252	260.0	2126.5	2212.1	4.6031	312.6	5.08	325.3	1
650.0	22.719	260.0	2136.2	2223.6	4.6181	169.1	4.65	338.8	1
655.0	23.873	260.0	2158.1	2249.9	4.6515	78.5	4.15	360.3	1
660.0	25.017	260.0	2178.1	2274.4	4.6820	51.1	3.89	376.2	1
665.0	26.156	260.0	2197.1	2297.7	4.7107	38.0	3.71	389.8	1
670.0	27.291	260.0	2215.3	2320.3	4.7379	30.4	3.57	401.9	1
675.0	28.423	260.0	2232.9	2342.2	4.7641	25.4	3.47	413.1	1
680.0	29.553	260.0	2250.0	2363.7	4.7894	21.9	3.38	423.6	1
685.0	30.682	260.0	2266.7	2384.7	4.8139	19.4	3.31	433.6	1
690.0	31.810	260.0	2283.1	2405.5	4.8377	17.4	3.25	443.1	1
695.0	32.936	260.0	2299.2	2425.9	4.8610	15.8	3.19	452.3	1

Table C1. Thermodynamic properties along isochores at regular temperature increments (continued).

Temp.	Pressure	Density	Internal Energy	Enthalpy	Entropy	C_p	C_v	Velocity of Sound	Phase Region
K	MPa	kg/m^3	kJ/kg	kJ/kg	kJ/kg.K	kJ/kg.K		m/s	
643.0	20.995	280.0	2033.8	2108.8	4.4502		11.42		2
644.0	21.247	280.0	2045.4	2121.3	4.4682		11.77		2
645.0	21.504	280.0	2057.4	2134.2	4.4868		12.31		2
646.0	21.764	280.0	2070.1	2147.9	4.5066		13.29		2
647.0	22.027	280.0	2083.8	2162.5	4.5278	2431.8	6.46	284.7	1
648.0	22.276	280.0	2089.7	2169.2	4.5368	635.3	5.42	307.6	1
650.0	22.770	280.0	2099.8	2181.1	4.5524	251.7	4.80	326.4	1
655.0	23.995	280.0	2122.0	2207.7	4.5865	96.1	4.20	352.2	1
660.0	25.215	280.0	2142.3	2232.3	4.6172	58.6	3.91	370.2	1
665.0	26.433	280.0	2161.3	2255.7	4.6460	42.1	3.72	385.2	1
670.0	27.651	280.0	2179.6	2278.3	4.6734	33.0	3.58	398.4	1
675.0	28.869	280.0	2197.2	2300.3	4.6996	27.2	3.48	410.5	1
680.0	30.088	280.0	2214.4	2321.8	4.7249	23.3	3.39	421.8	1
685.0	31.306	280.0	2231.1	2342.9	4.7495	20.4	3.31	432.4	1
690.0	32.525	280.0	2247.5	2363.7	4.7733	18.2	3.25	442.6	1
695.0	33.744	280.0	2263.7	2384.2	4.7966	16.5	3.19	452.3	1
643.0	20.995	300.0	2000.3	2070.3	4.3904		10.91		2
644.0	21.247	300.0	2011.4	2082.2	4.4076		11.24		2
645.0	21.504	300.0	2022.9	2094.5	4.4254		11.74		2
646.0	21.764	300.0	2035.0	2107.6	4.4442		12.66		2
647.0	22.028	300.0	2049.0	2122.4	4.4658		17.55		2
648.0	22.288	300.0	2055.9	2130.1	4.4764	1126.5	5.66	292.6	1
650.0	22.804	300.0	2066.2	2142.2	4.4924	326.8	4.86	316.9	1
655.0	24.095	300.0	2088.6	2168.9	4.5267	107.4	4.21	346.8	1
660.0	25.389	300.0	2108.8	2193.4	4.5574	62.7	3.91	366.7	1
665.0	26.686	300.0	2127.8	2216.8	4.5861	44.2	3.71	383.0	1
670.0	27.987	300.0	2146.0	2239.3	4.6134	34.2	3.58	397.2	1
675.0	29.291	300.0	2163.6	2261.3	4.6396	28.0	3.47	410.2	1
680.0	30.598	300.0	2180.7	2282.7	4.6648	23.8	3.38	422.2	1
685.0	31.907	300.0	2197.5	2303.8	4.6893	20.8	3.31	433.5	1
690.0	33.218	300.0	2213.8	2324.6	4.7131	18.5	3.24	444.2	1
695.0	34.531	300.0	2229.9	2345.0	4.7364	16.7	3.19	454.5	1
643.0	20.995	320.0	1971.1	2036.7	4.3380		10.47		2
644.0	21.247	320.0	1981.7	2048.1	4.3545		10.78		2
645.0	21.504	320.0	1992.7	2059.9	4.3716		11.24		2
646.0	21.764	320.0	2004.3	2072.3	4.3896		12.10		2
647.0	22.028	320.0	2017.6	2086.4	4.4102		16.69		2
648.0	22.295	320.0	2025.0	2094.7	4.4216	1380.9	5.66	283.5	1
650.0	22.832	320.0	2035.3	2106.6	4.4375	345.1	4.82	311.4	1
655.0	24.187	320.0	2057.4	2133.0	4.4714	107.5	4.16	344.4	1
660.0	25.555	320.0	2077.4	2157.3	4.5019	62.1	3.87	366.0	1
665.0	26.932	320.0	2096.3	2180.5	4.5303	43.7	3.68	383.5	1
670.0	28.318	320.0	2114.4	2202.9	4.5574	33.8	3.55	398.7	1
675.0	29.709	320.0	2131.8	2224.7	4.5834	27.7	3.45	412.4	1
680.0	31.107	320.0	2148.8	2246.1	4.6085	23.5	3.36	425.1	1
685.0	32.509	320.0	2165.5	2267.1	4.6328	20.6	3.29	437.0	1
690.0	33.915	320.0	2181.8	2287.7	4.6565	18.3	3.23	448.3	1
695.0	35.325	320.0	2197.8	2308.2	4.6796	16.6	3.17	459.0	1
643.0	20.995	340.0	1945.2	2007.0	4.2919		10.07		2
644.0	21.247	340.0	1955.4	2017.9	4.3077		10.37		2
645.0	21.504	340.0	1966.0	2029.2	4.3241		10.81		2
646.0	21.764	340.0	1977.2	2041.2	4.3414		11.61		2
647.0	22.028	340.0	1989.9	2054.7	4.3612		15.93		2
648.0	22.302	340.0	1996.7	2062.3	4.3716	1026.2	5.42	282.2	1
650.0	22.861	340.0	2006.6	2073.9	4.3869	289.0	4.67	310.8	1
655.0	24.283	340.0	2028.2	2099.7	4.4200	96.1	4.08	345.7	1
660.0	25.727	340.0	2047.9	2123.6	4.4499	57.0	3.81	368.6	1
665.0	27.189	340.0	2066.5	2146.4	4.4780	40.6	3.63	387.1	1
670.0	28.662	340.0	2084.3	2168.6	4.5047	31.8	3.51	403.1	1
675.0	30.146	340.0	2101.6	2190.3	4.5304	26.2	3.41	417.5	1
680.0	31.638	340.0	2118.4	2211.5	4.5553	22.5	3.33	430.8	1
685.0	33.137	340.0	2134.9	2232.4	4.5794	19.7	3.26	443.3	1
690.0	34.643	340.0	2151.1	2253.0	4.6030	17.6	3.21	455.0	1
695.0	36.154	340.0	2167.0	2273.3	4.6259	16.0	3.15	466.2	1

Table C1. Thermodynamic properties along isochores at regular temperature increments (continued).

Temp.	Pressure	Density	Internal Energy	Enthalpy	Entropy	C_p	C_v	Velocity of Sound	Phase Region
K	MPa	kg/m^3	kJ/kg	kJ/kg	kJ/kg.K	kJ/kg.K		m/s	
643.0	20.995	360.0	1922.3	1980.6	4.2508		9.73		2
644.0	21.247	360.0	1932.1	1991.1	4.2661		10.00		2
645.0	21.504	360.0	1942.3	2002.0	4.2819		10.42		2
646.0	21.764	360.0	1953.1	2013.5	4.2986		11.18		2
647.0	22.029	360.0	1965.0	2026.2	4.3171	2889.7	6.16	253.2	1
648.0	22.314	360.0	1970.4	2032.4	4.3255	520.9	5.01	289.5	1
650.0	22.898	360.0	1979.8	2043.4	4.3399	199.5	4.46	316.1	1
655.0	24.395	360.0	2000.7	2068.4	4.3719	78.1	3.96	351.3	1
660.0	25.924	360.0	2019.8	2091.9	4.4010	48.9	3.73	375.0	1
665.0	27.475	360.0	2038.1	2114.4	4.4285	36.0	3.57	394.2	1
670.0	29.043	360.0	2055.6	2136.3	4.4548	28.7	3.46	410.8	1
675.0	30.625	360.0	2072.7	2157.7	4.4802	24.0	3.37	425.8	1
680.0	32.217	360.0	2089.3	2178.8	4.5048	20.8	3.29	439.6	1
685.0	33.820	360.0	2105.6	2199.6	4.5287	18.4	3.23	452.5	1
690.0	35.430	360.0	2121.7	2220.1	4.5520	16.6	3.18	464.7	1
695.0	37.047	360.0	2137.4	2240.3	4.5748	15.2	3.13	476.3	1
643.0	20.995	380.0	1901.7	1957.0	4.2141		9.41		2
644.0	21.247	380.0	1911.3	1967.2	4.2289		9.68		2
645.0	21.504	380.0	1921.1	1977.7	4.2442		10.07		2
646.0	21.764	380.0	1931.5	1988.8	4.2603		10.79		2
647.0	22.037	380.0	1940.9	1998.9	4.2749	455.4	4.97	284.6	1
648.0	22.339	380.0	1945.7	2004.5	4.2822	232.8	4.57	304.7	1
650.0	22.955	380.0	1954.4	2014.8	4.2957	123.6	4.21	327.5	1
655.0	24.540	380.0	1974.4	2039.0	4.3263	59.3	3.83	361.6	1
660.0	26.163	380.0	1993.0	2061.9	4.3546	40.0	3.63	385.5	1
665.0	27.814	380.0	2010.8	2084.0	4.3815	30.6	3.50	405.1	1
670.0	29.485	380.0	2028.0	2105.6	4.4073	25.1	3.40	422.2	1
675.0	31.173	380.0	2044.8	2126.9	4.4323	21.4	3.32	437.6	1
680.0	32.874	380.0	2061.3	2147.8	4.4565	18.8	3.25	451.7	1
685.0	34.586	380.0	2077.4	2168.4	4.4801	16.8	3.20	465.0	1
690.0	36.309	380.0	2093.2	2188.8	4.5032	15.3	3.15	477.4	1
695.0	38.040	380.0	2108.9	2209.0	4.5258	14.1	3.10	489.3	1
643.0	20.995	400.0	1883.2	1935.7	4.1810		9.13		2
644.0	21.247	400.0	1892.5	1945.6	4.1954		9.38		2
645.0	21.504	400.0	1902.0	1955.8	4.2102		9.75		2
646.0	21.764	400.0	1912.1	1966.5	4.2258		10.44		2
647.0	22.072	400.0	1917.6	1972.8	4.2343	150.3	4.35	313.1	1
648.0	22.395	400.0	1921.8	1977.8	4.2409	110.4	4.18	326.2	1
650.0	23.052	400.0	1930.0	1987.6	4.2534	74.8	3.97	345.1	1
655.0	24.741	400.0	1949.0	2010.9	4.2827	43.7	3.69	377.1	1
660.0	26.473	400.0	1967.1	2033.3	4.3101	31.9	3.53	400.6	1
665.0	28.235	400.0	1984.5	2055.1	4.3363	25.5	3.42	420.2	1
670.0	30.020	400.0	2001.4	2076.4	4.3616	21.5	3.34	437.5	1
675.0	31.824	400.0	2017.9	2097.4	4.3862	18.8	3.27	453.0	1
680.0	33.642	400.0	2034.1	2118.2	4.4101	16.7	3.21	467.4	1
685.0	35.474	400.0	2050.0	2138.7	4.4334	15.2	3.16	480.8	1
690.0	37.317	400.0	2065.7	2158.9	4.4562	14.0	3.11	493.5	1
695.0	39.169	400.0	2081.1	2179.0	4.4785	12.9	3.07	505.5	1
643.0	20.995	420.0	1866.5	1916.5	4.1511		8.88		2
644.0	21.247	420.0	1875.5	1926.1	4.1651		9.12		2
645.0	21.504	420.0	1884.8	1936.0	4.1795		9.47		2
646.0	21.819	420.0	1890.5	1942.5	4.1884	85.9	4.08	331.9	1
647.0	22.164	420.0	1894.6	1947.3	4.1947	69.7	3.97	343.4	1
648.0	22.513	420.0	1898.5	1952.1	4.2007	59.5	3.88	352.9	1
650.0	23.220	420.0	1906.1	1961.4	4.2125	47.0	3.76	368.5	1
655.0	25.033	420.0	1924.4	1984.0	4.2405	32.2	3.56	397.7	1
660.0	26.889	420.0	1941.8	2005.9	4.2670	25.2	3.44	420.4	1
665.0	28.776	420.0	1958.8	2027.3	4.2926	21.1	3.35	439.6	1
670.0	30.687	420.0	1975.4	2048.4	4.3174	18.3	3.28	456.7	1
675.0	32.617	420.0	1991.6	2069.2	4.3416	16.3	3.22	472.2	1
680.0	34.565	420.0	2007.5	2089.8	4.3651	14.8	3.17	486.6	1
685.0	36.526	420.0	2023.3	2110.2	4.3882	13.6	3.12	500.1	1
690.0	38.499	420.0	2038.8	2130.4	4.4107	12.6	3.08	512.8	1
695.0	40.482	420.0	2054.1	2150.5	4.4328	11.8	3.04	525.0	1

Table C2. Properties of coexisting phases at regular temperature increments.

vapor

Temp.	Pressure	Density	Latent Heat	Internal Energy	Enthalpy	Entropy	C_p	C_v	Velocity of Sound
K	MPa	kg/m^3	kJ/kg	kJ/kg	kJ/kg	kJ/kg.K	kJ/kg.K		m/s
643.00	20.995	199.78	451.8	2235.3	2340.4	4.8104	95.4	4.64	361.1
643.20	21.045	201.78	443.3	2231.2	2335.5	4.8024	100.4	4.67	359.6
643.40	21.095	203.84	434.5	2227.0	2330.5	4.7943	106.0	4.70	358.0
643.60	21.146	205.99	425.5	2222.7	2325.3	4.7858	112.2	4.73	356.4
643.80	21.197	208.22	416.2	2218.2	2320.0	4.7771	119.3	4.76	354.7
644.00	21.247	210.55	406.5	2213.5	2314.4	4.7681	127.3	4.80	352.9
644.20	21.298	212.97	396.5	2208.6	2308.6	4.7588	136.6	4.83	350.9
644.40	21.350	215.52	386.1	2203.6	2302.6	4.7491	147.3	4.88	348.9
644.60	21.401	218.20	375.2	2198.2	2296.3	4.7389	159.8	4.92	346.7
644.80	21.452	221.03	363.8	2192.7	2289.7	4.7283	174.7	4.97	344.4
645.00	21.504	224.04	351.8	2186.8	2282.7	4.7171	192.7	5.02	341.8
645.20	21.555	227.25	339.1	2180.5	2275.3	4.7053	214.7	5.08	339.1
645.40	21.607	230.70	325.6	2173.8	2267.5	4.6927	242.5	5.15	336.0
645.60	21.659	234.44	311.1	2166.6	2259.0	4.6792	278.3	5.23	332.7
645.80	21.712	238.55	295.3	2158.7	2249.7	4.6646	326.2	5.32	328.8
646.00	21.764	243.12	277.9	2150.1	2239.6	4.6485	393.6	5.42	324.4
646.20	21.816	248.31	258.5	2140.3	2228.2	4.6305	494.6	5.55	319.1
646.40	21.869	254.39	236.0	2129.0	2215.0	4.6098	661.6	5.72	312.6
646.60	21.922	261.87	208.7	2115.3	2199.1	4.5849	986.2	5.96	304.0
646.80	21.975	271.99	172.6	2097.2	2178.0	4.5521	1858.6	6.36	291.0
647.00	22.028	290.37	108.7	2065.6	2141.5	4.4953	9198.8	7.47	261.7
647.067	22.0460	322.778	.0	2014.8	2083.1	4.4050			.0

liquid

Temp.	Pressure	Density	Latent Heat	Internal Energy	Enthalpy	Entropy	C_p	C_v	Velocity of Sound
K	MPa	kg/m^3	kJ/kg	kJ/kg	kJ/kg	kJ/kg.K	kJ/kg.K		m/s
~~645.00~~	~~21.504~~	~~425.44~~	~~351.8~~	~~1880.4~~	~~1930.9~~	~~4.1716~~	~~88.2~~	~~4.11~~	~~328.6~~
645.20	21.555	421.90	339.1	1885.1	1936.2	4.1797	98.7	4.17	324.0
645.40	21.607	418.11	325.6	1890.2	1941.8	4.1882	112.1	4.23	319.2
645.60	21.659	414.02	311.1	1895.6	1947.9	4.1974	129.7	4.30	314.0
645.80	21.712	409.57	295.3	1901.4	1954.4	4.2074	153.6	4.39	308.5
646.00	21.764	404.64	277.9	1907.9	1961.6	4.2183	187.8	4.50	302.6
646.20	21.816	399.08	258.5	1915.0	1969.7	4.2306	240.3	4.63	295.9
646.40	21.869	392.63	236.0	1923.3	1979.0	4.2448	329.7	4.80	288.4
646.60	21.922	384.76	208.7	1933.4	1990.3	4.2621	510.2	5.06	279.2
646.80	21.975	374.23	172.6	1946.7	2005.5	4.2852	1024.2	5.48	267.0
647.00	22.028	355.38	108.7	1970.8	2032.8	4.3272	5920.7	6.69	243.0
647.067	22.0460	322.778	.0	2014.8	2083.1	4.4050			.0

Table C3. Properties of coexisting phases at regular pressure intervals.

vapor

Temp.	Pressure	Density	Latent Heat	Internal Energy	Enthalpy	Entropy	C_p	C_v	Velocity of Sound
K	MPa	kg/m^3	kJ/kg	kJ/kg	kJ/kg	kJ/kg.K	kJ/kg.K		m/s
643.02	21.000	199.99	450.9	2234.9	2339.9	4.8096	95.9	4.64	361.0
643.22	21.050	201.98	442.4	2230.8	2335.0	4.8016	101.0	4.67	359.5
643.42	21.100	204.04	433.7	2226.6	2330.0	4.7935	106.5	4.70	357.9
643.62	21.150	206.17	424.8	2222.3	2324.9	4.7851	112.8	4.73	356.3
643.81	21.200	208.37	415.6	2217.9	2319.6	4.7765	119.8	4.76	354.6
644.01	21.250	210.66	406.0	2213.3	2314.1	4.7677	127.8	4.80	352.8
644.21	21.300	213.05	396.2	2208.5	2308.5	4.7585	136.9	4.84	350.9
644.40	21.350	215.54	386.0	2203.5	2302.6	4.7490	147.4	4.88	348.9
644.60	21.400	218.16	375.4	2198.3	2296.4	4.7391	159.6	4.92	346.7
644.79	21.450	220.91	364.3	2192.9	2290.0	4.7288	174.0	4.97	344.5
644.99	21.500	223.81	352.7	2187.2	2283.3	4.7180	191.2	5.02	342.0
645.18	21.550	226.90	340.5	2181.2	2276.1	4.7066	212.1	5.08	339.4
645.37	21.600	230.19	327.6	2174.8	2268.6	4.6946	238.1	5.14	336.5
645.56	21.650	233.74	313.8	2167.9	2260.5	4.6817	271.1	5.21	333.3
645.76	21.700	237.60	298.9	2160.5	2251.9	4.6680	314.3	5.29	329.7
645.95	21.750	241.86	282.7	2152.4	2242.4	4.6530	373.3	5.39	325.6
646.14	21.800	246.62	264.8	2143.5	2231.9	4.6364	458.2	5.51	320.9
646.33	21.850	252.07	244.5	2133.3	2220.0	4.6177	590.2	5.65	315.2
646.52	21.900	258.56	220.7	2121.4	2206.1	4.5959	821.4	5.85	307.9
646.71	21.950	266.77	191.1	2106.5	2188.8	4.5689	1320.1	6.14	297.9
646.89	22.000	278.71	149.0	2085.5	2164.4	4.5309	3066.8	6.69	281.3
647.067	22.0460	322.778	.0	2014.8	2083.1	4.4050			.0

liquid

Temp.	Pressure	Density	Latent Heat	Internal Energy	Enthalpy	Entropy	C_p	C_v	Velocity of Sound
K	MPa	kg/m^3	kJ/kg	kJ/kg	kJ/kg	kJ/kg.K	kJ/kg.K		m/s
644.99	21.500	425.69	352.7	1880.0	1930.5	4.1711	87.5	4.11	328.9
645.18	21.550	422.28	340.5	1884.6	1935.6	4.1788	97.5	4.16	324.5
645.37	21.600	418.66	327.6	1889.4	1941.0	4.1870	110.0	4.22	319.9
645.56	21.650	414.78	313.8	1894.6	1946.8	4.1957	126.1	4.29	315.0
645.76	21.700	410.59	298.9	1900.1	1952.9	4.2051	147.6	4.37	309.8
645.95	21.750	406.00	282.7	1906.1	1959.7	4.2153	177.4	4.47	304.2
646.14	21.800	400.89	264.8	1912.7	1967.1	4.2266	221.3	4.58	298.1
646.33	21.850	395.08	244.5	1920.2	1975.5	4.2394	291.2	4.74	291.2
646.52	21.900	388.23	220.7	1928.9	1985.4	4.2545	417.6	4.94	283.3
646.71	21.950	379.64	191.1	1939.9	1997.7	4.2733	702.9	5.25	273.3
646.89	22.000	367.29	149.0	1955.5	2015.4	4.3006	1777.1	5.84	258.6
647.067	22.0460	322.778	.0	2014.8	2083.1	4.4050			.0

Appendix D. Computer Program for Table Generation

In this Appendix the FORTRAN program is listed that yields the thermodynamic properties of steam in the region shown in Fig. 1 and specified by Eq. (18).

The independent variables are temperature T in K and density D in kg/m^3. The program computes the following properties: presssure P in MPa, energy U in kJ/kg, enthalpy H in kJ/kg, entropy S in kJ/kg.K, specific heat at constant volume CV in kJ/kg.K, specific heat at constant pressure CP in kJ/kg.K, velocity of sound CS in m/s, compressibility COMP in 1/MPa.

The transformation from temperature and density to the parametric variables r and θ of the extended and revised scaled equations is performed iteratively in the subroutine CONVER. Initial guesses for r and θ to be used in CONVER are provided by the subroutine RTHETA of Moldover[58] which solves simple scaled equations with $c = 0$ and $k_1 = 0$. The pressure and other thermodynamic properties are calculated in the subroutine THERMO.

As a second option the temperature and pressure can be used as input variables. In this case, first the density is found from the equation of state by Newton iteration in the subroutine DFIND, after which the computations proceed as described above.

If saturation properties are desired, the independent variable is temperature; in this case $\theta = -1(+1)$ for the vapor (liquid) side and $r = \Delta\widetilde{T}/(1 - b^2)$. All properties, including density and latent heat, are then calculated in THERMO. As a second option the pressure can be used as the input variable. The temperature is then found from the vapor pressure equation in the subroutine TFIND after which the calculation proceeds as described above.

The subroutine RANGE checks whether the desired point lies within the range of the validity of the equation of state.

An example of the usage of these subroutines is given by the main program shown and a sample of the output is presented.

This code has been developed and tested on a Univac 1180/1108 computer.

```
--------------------------------------------------------------------------------
      BLOCKDATA
C
C  THIS SUBROUTINE SUPPLIES THE PARAMETERS USED IN THE EQUATION
C  OF STATE.
C
      DOUBLE PRECISION A,Q
      COMMON /COEFS/A(20),Q(20),ANAMES(20),QNAMES(20)
      DATA A/-.017762D0,5.2380D0,0.D0,-25.4915D0,6.8445D0,.325D0,1.4403
     1D0,0.D0,1.3757D0,23.6666D0,4.82D0,.2942D0,-11.2326D0,-22.6547D0
     2,-17.8876D0,-4.9332D0,4*0.D0/
      DATA Q/-.006D0,-.003D0,0.D0,647.067D0,322.778D0,22.0460D0
     1,.267D0,-1.6D0,12*0.D0/
      DATA ANAMES/6HC     ,6HP3    ,6HDELROC,6HP2    ,6HP1    ,6HBETA
     1,6HK0    ,6HDELTC ,6HB*B   ,6HA     ,6HDELTA ,6HK1    ,6HMUC
     2,6HMU1   ,6HMU2   ,6HMU3   ,6HS00   ,6HS20   ,6HS01   ,6HS21   /
      DATA QNAMES/6HPOINTA,6HPOINTB,6HDELPC ,6HTC    ,6HRHOC  ,6HPC
     1,6HDPCDTC,6HSLOPDI,6HP11   ,6HALPHA ,6HP00   ,6HP20   ,6HP40
     2,6HDELTAI,6HALPHAI,6HBETA I,6HGAMMAI,6HP01   ,6HP21   ,6HP41   /
      END
--------------------------------------------------------------------------------
      SUBROUTINE CONST
C
C THIS SUBROUTINE CALCULATES ALL QUANTITIES NOT DEPENDENT ON R OR THETA.
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON /COEFS/ A(20),Q(20)
      COMMON/CRITS/TC,RHOC,PC,PCON,UCON,SCON,DPCON
      EQUIVALENCE (DELI,Q(14)),(BETA,A(6)),(DELTA,A(11)),(BESQ,A(9))
     1,(CC,A(1)),(AA,A(10)),(XK0,A(7)),(ALPHA,Q(10))
     2,(ALHI,Q(15)),(BETI,Q(16)),(GAMI,Q(17)),(P01,Q(18)),(P21,Q(19))
     3,(P00,Q(11)),(P20,Q(12)),(P40,Q(13)),(P41,Q(20))
     4,(S00,A(17)),(S20,A(18)),(S01,A(19)),(S21,A(20))
C
      ALPHA = 2.D0 - A(6)*(A(11) + 1.D0)
      GAMMA = BETA*(DELTA - 1.D0)
      DELI =  0.50D0
      ALHI = ALPHA - DELI
      BETI = BETA + DELI
      GAMI = GAMMA - DELI
      ERR = 2.D0*BETA*DELTA - 1.D0
      P00 = (BETA*(DELTA-3.D0)-BESQ*ALPHA*GAMMA)/(2.D0*
     1 BESQ*BESQ*(2.D0-ALPHA)*(1.D0-ALPHA)*ALPHA)
      P20=-(BETA*(DELTA-3.)-BESQ*ALPHA*ERR)/(2.*BESQ*(1.-ALPHA)*(ALPHA))
      P40=(ERR-2.)/2./ALPHA
      S00 = (2.-ALPHA)*P00
      S20 = -BETA*(DELTA-3.)/2./BESQ/ALPHA
      P01 = (BETA*(DELTA-3.D0)-3.D0*DELI-A(9)*ALHI*GAMI)
     1 /(2.D0*A(9)*A(9)*(2.D0-ALHI)*(1.D0-ALHI)*ALHI)
      P21 = -((BETA*(DELTA-3.D0)-3.D0*DELI-A(9)*ALHI*ERR)
     1  (2.D0*A(9)*(1.D0-ALHI)*ALHI))
      P41 = (.5*ERR - 1.)/ALHI
      S01 = (2.-ALHI)*P01
      S21 = -(BETA*DELTA-3.*BETI)/2./BESQ/ALHI
C
C  CALCULATE P11 FROM THE DIAMETER OF THE COEXISTENCE CURVE.
      DA=Q(1)
      DB=Q(2)
      RA = DA/(1.D0 - BESQ)
      RB = DB/(1.D0 - BESQ)
      SW0 = S00+S20
      DR0=SW0*(RA**(1.-ALPHA)-RB**(1.-ALPHA))
      PTW = P01 + P21 + P41
      SW1 = S01+S21
      DR1=SW1*(RA**(1.-ALHI)-RB**(1.-ALHI))
      D1=CC*AA*XK0*DR0
      D2=CC*AA*A(12)*DR1
      Q(9) = (Q(8)*(1./(1.-DB)-1./(1.-DA))+D1+D2)/(DB-DA)
C
      Q(3)=A(8)*Q(7)
      TC=Q(4)+A(8)
      RHOC=Q(5)+A(3)
      PC=Q(6)+Q(3)
      PCON=PC/TC
      UCON=1.D3*PC
      SCON=UCON/TC
      DPCON=PCON/RHOC/RHOC
      RETURN
      END
--------------------------------------------------------------------------------
      SUBROUTINE RANGE(IRANGE,IOPT,T,D,P)
C
C  THIS ROUTINE DETERMINES WHETHER THE ENTRY POINT IS WITHIN THE RANGE
C  OF VALIDITY OF THE EQUATION OF STATE (IRANGE=1), OR NOT (IRANGE=0).
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SATUR/ISAT
```

```
      COMMON/CRITS/TC,RHOC,PC
      DATA TMIN,TMAX/643.D0,695.D0/,DMIN,DMAX/200.D0,420.D0/
C
      IRANGE=1
      IF(ISAT.NE.0) GO TO 10
      IF(T.LT.TMIN .OR. T.GT.TMAX) GO TO 99
      IF(IOPT.EQ.2) GO TO 20
      IF(D.LT.DMIN .OR. D.GT.DMAX) GO TO 99
      RETURN
 20   CALL THERMO(1,T,DMIN,PMIN,R,THETA)
      CALL THERMO(1,T,DMAX,PMAX,R,THETA)
      IF(P.LT.PMIN .OR. P.GT.PMAX) GO TO 99
      RETURN
 10   IF(IOPT.EQ.2) GO TO 30
      IF(T.GE.TC) GO TO 99
      IF(ISAT.EQ.-1 .AND. T.LT.TMIN) GO TO 99
      IF(ISAT.EQ. 1 .AND. T.LT.645.D0) GO TO 99
      RETURN
 30   IF(P.GE.PC) GO TO  99
      IF(ISAT.EQ.-1 .AND. P.LT.21.0D0) GO TO 99
      IF(ISAT.EQ. 1 .AND. P.LT.21.5D0) GO TO 99
      RETURN
 99   IRANGE=0
      WRITE(6,2)
 2    FORMAT(2X,'THIS POINT IS OUTSIDE THE RANGE!'/)
      RETURN
      END
-------------------------------------------------------------------------------
      SUBROUTINE THERMO(IDFIND,T,D,P,R1,TH1)
C
C  1. NOT ON THE SATURATION CURVE (ISAT=0) :
C  GIVEN THE TEMPERATURE T(K) AND DENSITY D(KG/M3), THIS ROUTINE
C  CALCULATES PRESSURE P(MPA) AND ITS DERIVATIVES W.R.T D AT
C  CONSTANT T DPDD(MPA/(KG/M3)) AND W.R.T T AT CONSTANT D
C  DPDT(MPA/K), ENERGY U AND ENTHALPY H(KJ/KG), ENTROPY AND
C  SPECIFIC HEATS CP AND CV(KJ/KGK), VELOCITY OF SOUND CS(M/S),
C  AND COMPRESSIBILITY COMP(1/MPA).
C  IF THE ENTRY POINT IS IN THE 2-PHASE REGION (IPHASE=2) IT
C  RETURNS CP=COMP=CS=0.
C  IF IDFIND=1 IT CALCULATES ONLY P AND DPDD.
C  2. ON THE SATURATION CURVE (ISAT.NE.0) :
C  GIVEN THE TEMPERATURE T, IT CALCULATES THE DENSITY D, LATENT HEAT
C  AND ALL THE ABOVE PROPERTIES ON THE VAPOR SIDE (ISAT=-1), OR THE
C  LIQUID SIDE (ISAT=1).
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON/THERM/IPHASE,DPDD,DPDT,U,H,ENTROP,CP,CV,CS,COMP
      COMMON/SATUR/ISAT,HEAT,DLIQ,DVAP
      COMMON/COEFS/A(20),Q(20)
      COMMON/CRITS/TC,RHOC,PC,PCON,UCON,SCON,DPCON
      DIMENSION S(2),XK(2),SD(2)
      EQUIVALENCE (PW1,A(5)),(PW2,A(4)),(PW3,A(2)),
     1   (AMC,A(13)),(AM1,A(14)),(AM2,A(15)),(AM3,A(16)),
     2   (P00,Q(11)),(P20,Q(12)),(P40,Q(13)),
     3   (P01,Q(18)),(P21,Q(19)),(P41,Q(20)),
     4   (AA,A(10)),(XK0,A(7)),(XK1,A(12)),(PW11,Q(9)),
     5   (ALPHA,Q(10)),(ALHI,Q(15)),(BESQ,A(9))
C
      XK(1)=XK0
      XK(2)=XK1
      TEE=(T-TC)/TC
      TW=-TC/T
      DTW=1.D0+TW
      IF(ISAT.NE.0) GO TO 10
      RHO=D/RHOC
      CALL CONVER(RHO,TEE,AMU,TH1,R1,RHO1,S,RHODI,ERR)
      GO TO 12
 10   TH1=1.D0
      IF(ISAT.EQ.-1) TH1=-0.99999999999999999D0
      IF(ISAT.EQ. 1) TH1= 0.99999999999999999D0
      R1=DTW/(1.D0-BESQ)
      CALL SS(R1,TH1,S,SD)
      RHO=TH1*(XK0*R1**A(6)+XK1*R1**Q(16))+A(1)*(S(1)+S(2''
      RHO=1.D0+PW11*DTW+RHO
      D=RHO*RHOC
      AMU=0.D0
 12   TT1=TH1*TH1
      TT2=TT1*TT1
      PW0=1.D0+DTW*(PW1+DTW*(PW2+DTW*PW3))
      PWMU=AMU*RHODI
      P0TH=P00+P20*TT1+P40*TT2
      P1TH=P01+P21*TT1+P41*TT2
      DPW0=XK0*P0TH*R1**(2.-ALPHA)
      DPW1=XK1*P1TH*R1**(2.-ALHI)
      DPW=AA*(DPW0+DPW1)
      PW=PW0+PWMU+DPW
      IF(IDFIND.EQ.1) GO TO 11
```

```
      DPODT=PW1+DTW*(2.DO*PW2+3.DO*PW3*DTW)
      DMODT=AM1+DTW*(2.DO*AM2+3.DO*AM3*DTW)
      UW=DPODT-RHO*DMODT+PW11*AMU+S(1)+S(2)
      HW=PW-TW*UW
      AMW=AMU+AMC+DTW*(AM1+DTW*(AM2+DTW*AM3))
      SW=HW-RHO*AMW
C
      D2PODT=2.DO*PW2+6.DO*PW3*DTW
      D2MODT=2.DO*AM2+6.DO*AM3*DTW
 11   IF(DABS(TH1).GE.1.DO) GO TO 13
      IPHASE=1
      CALL AUX(R1,TH1,D2PDT2,D2PDMT,D2PDM2,AA,XK,SD,CVCOEX)
      DPDD=DPCON*D*T/D2PDM2
      IF(IDFIND.EQ.1) GO TO 15
      DPDTCD=DPODT+PW11*(AMU-RHO/D2PDM2)+S(1)+S(2)-D2PDMT*RHO/D2PDM2
      DPWDTW=PW-TW*DPDTCD
      CVITW2=D2PODT-RHO*D2MODT+D2PDT2-(PW11+D2PDMT)**2/D2PDM2
      CVW=CVITW2*TW*TW
      CPW=CVW+D2PDM2*DPWDTW*DPWDTW/(RHO*RHO)
      COMP=1.DO/(D*DPDD)
      CS=1.D3*DSQRT(CPW/CVW*DPDD)
      IF(ISAT.EQ.0) GO TO 14
      RHO1=1.DO+PW11*DTW+A(1)*(S(1)+S(2))
      RHO2=XKO*R1**A(6)+XK1*R1**Q(16)
      DLIQ=RHOC*(RHO1+RHO2)
      DVAP=RHOC*(RHO1-RHO2)
      DPDT2=PW-TW*(UW+RHO*DMODT)
      HEAT=1.D3*T*(PCON*DPDT2)*(1.DO/DVAP-1.DO/DLIQ)
      GO TO 14
 13   IPHASE=2
      DPDD=0.
      IF(IDFIND.EQ.1) GO TO 15
      DPDTCD=UW+RHO*DMODT
      DPWDTW=PW-TW*DPDTCD
      CVIO=(2.-ALPHA)*(1.-ALPHA)*R1**(-ALPHA)*XKO*POTH
      CVI1=(2.-ALHI )*(1.-ALHI )*R1**(-ALHI )*XK1*P1TH
      CVITW2=D2PODT-RHO*D2MODT+AA*(1./(1.-BESQ))**2*(CVIO+CVI1)
      CVW=CVITW2*TW*TW
      CPW=0.
      COMP=0.
      CS=0.
C
 14   DPDT=PCON*DPWDTW
      SCOND=SCON/D
      U=UW*UCON/D
      H=HW*SCOND*T
      ENTROP=SW*SCOND
      CV=CVW*SCOND
      CP=CPW*SCOND
 15   P=PW*PCON*T
      RETURN
      END
----------------------------------------------------------------------------
      SUBROUTINE DFIND(T,P,D,DGUESS,DMIN,DMAX)
C
C  GIVEN THE TEMPERATURE T(K), PRESSURE P(MPA), AND AN INITIAL
C  GUESS DENSITY DGUESS(KG/M3), THIS ROUTINE FINDS THE
C  CORRESPONDING DENSITY D(KG/M3) IN THE RANGE (DMIN,DMAX).
C  IF T IS BELOW THE CRITICAL ISOTHERM, DGUESS MUST BE SET
C  EQUAL TO THE CRITICAL DENSITY, OTHERWISE IT MAY YIELD
C  ERRONEOUS RESULTS. IF THE ENTRY POINT IS IN THE 2-PHASE
C  REGION CRITICAL DENSITY IS RETURNED FOR D.
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/THERM/IPHASE,DPDD
C
      D=DGUESS
      DO 10 I=1,20
      CALL THERMO(1,T,D,PP,R,THETA)
      PDIF=PP-P
      IF(IPHASE.EQ.2) GO TO 18
      DELD=-PDIF/DPDD
      D=D+DELD
      IF(D.LT.DMIN) D=DMIN
      IF(D.GT.DMAX) D=DMAX
      IF(DABS(DELD/D).LT.1.D-6) GO TO 19
      GO TO 10
 18   IF(DABS(PDIF).LE.0.DO) GO TO 19
      IF(PDIF.LT.0.DO) D=DMAX
      IF(PDIF.GT.0.DO) D=DMIN
 10   CONTINUE
      WRITE(6,21) T,P,PP,D
 21   FORMAT(5X,'DFIND DOES NOT CONVERGE :',4F12.4)
 19   RETURN
      END
----------------------------------------------------------------------------
```

```
      SUBROUTINE TFIND(P,T)
C
C  GIVEN A PRESSURE P(MPA) BELOW THE CRITICAL ISOBAR, IT FINDS THE
C  CORRESPONDING TEMPERATURE T(K) ON THE SATURATION CURVE.
C
      IMPLICIT DOUBLE PRECISION (A-H,O-Z)
      COMMON/THERM/IPHASE,DPDD,DPDT,U,H,ENTROP,CP,CV,CS,COMP
      COMMON/SATUR/ISAT,HEAT,RHOL,RHOG
      COMMON/CRITS/TC,RHOC,PC,PCON,UCON,SCON,DPCON
C
      T=TC-1.D0
      D=RHOC
      ISAVE=ISAT
      ISAT=2
      DO 10 I=1,20
      CALL THERMO(0,T,D,PP,R1,TH1)
      DT=(PP-P)/DPDT
      T=T-DT
      IF(T.GT.TC) GO TO 12
      IF(DABS(DT/T).LT.1.D-8) GO TO 20
      GO TO 10
 12   T=TC-0.001D0
 10   CONTINUE
      WRITE(6,11) P,PP,T
 11   FORMAT(1X/1X'TFIND DOES NOT CONVERGE :',3F12.6/)
 20   ISAT=ISAVE
      RETURN
      END
```

```
      SUBROUTINE CONVER(RHO,TEE,AMU,TH1,R1,RHO1S,S1,RHODI,ERROR1)
C
C  THIS ROUTINE TRANSFORMS TEMPERATURE AND DENSITY TO THE PARAMETRIC VARIABLES
C  R AND THETA ACCORDING TO THE REVISED AND EXTENDED SCALED EQUATIONS.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /COEFS/ A(20),Q(20)
      DIMENSION S1(2),SD(2)
      EQUIVALENCE(BETA,A(6)),(DELTA,A(11)),(XK1,A(12)),(CC,A(1))
     1 ,(ALHI,Q(15)),(ALPHA,Q(10)),(BESQ,A(9)),(P11,Q(9))
     2 ,(DELI,Q(14)),(P1W,Q(18)),(P2W,Q(19)),(P4W,Q(20))
     3,(AA,A(10)),(XK0,A(7)),(S00,A(17)),(S20,A(18)),(BETAI,Q(16))
C
      TSTAR = TEE+1.D0
       DTSTIN = 1.D0 -  (1.D0/TSTAR)
      R1=DTSTIN
      IF(DTSTIN.LT.0.) R1=DTSTIN/(1.-BESQ)
      TH1=0.
      IF(DTSTIN.LT.0.) TH1=1.
      CALL SS(R1,TH1,S1,SD)
      RHODI = 1.D0 + P11*DTSTIN
      RHODIT = RHODI + CC*S1(1) + CC*S1(2)
      DRHO = RHO - RHODIT
      AMU = 0.D0
      IF(DTSTIN.GT.0.D0) GO TO 1
      RHO1CO = XK0*R1**BETA + XK1*R1**BETAI
      TWOFAZ = RHO1CO
      IF (DABS(DRHO).GT.TWOFAZ) GO TO 1
      RHO1S = DSIGN(RHO1CO,DRHO) + CC*S1(1)
      TH1 = DSIGN(1.D0,DRHO)
      ERROR1 = 1.D0
      GO TO 999
  1   CONTINUE
      IF(DRHO.NE.0.D0) GO TO 2
      TH1 = 0.D0
      R1 = DTSTIN
      RHO1S = CC*S1(1)
   2  CONTINUE
C RULE FOR FIRST PASS
       Y1 = DTSTIN
        DEN1 = RHO - RHODIT
      CALL RTHETA(R1,TH1,DEN1,Y1)
      TT=TH1*TH1
       AMU = AA* R1**(BETA*DELTA)*TH1*(1.D0-TT)
      Y1 = DTSTIN + CC*AMU
      CALL SS(R1,TH1,S1,SD)
        RHOWEG =  XK1*(R1**BETAI)*TH1 + CC*S1(2)
      RHO1S = DEN1 + CC*S1(1) + RHOWEG
      ERROR1 = RHO - RHODI - RHO1S
      IF( DABS(ERROR1).LT.1.D-5 ) GO TO 999
C RULE FOR SECOND PASS
      DEN12 = RHO - RHODI    - CC*S1(1) + RHOWEG
        IF (DEN12.EQ.DEN1) DEN12 = DEN1 - 1.D-06
      CALL RTHETA(R1,TH1,DEN12,Y1)
      TT = TH1*TH1
       AMU = AA* R1**(BETA*DELTA)*TH1*(1.D0-TT)
      Y1 = DTSTIN + CC*AMU
      CALL SS(R1,TH1,S1,SD)
        RHOWEG =  XK1*R1**BETAI*TH1 + CC*S1(2)
```

```
      RHO1S2 = DEN12 + CC*S1(1) + RHOWEG
      ERROR2 = RHO - RHODI - RHO1S2
      IF( DABS(ERROR2).GT.1.D-5 ) GO TO 998
      ERROR1 = ERROR2
      RHO1S = RHO1S2
      GO TO 999
 998  CONTINUE
C RULE FOR NTH PASS
       DEN2 = DEN12
      DO 44 ISIG = 1,10
      SLOPE = (ERROR2-ERROR1)/(DEN2 -DEN1)
        HOLD = DEN2
      DEN2 = DEN1 - (ERROR1/SLOPE)
      CALL RTHETA(R1,TH1,DEN2,Y1)
      TT = TH1*TH1
       AMU = AA* R1**(BETA*DELTA)*TH1*(1.D0-TT)
       Y1 = DTSTIN + CC*AMU
      CALL SS(R1,TH1,S1,SD)
        RHOWEG =  XK1*R1**BETAI*TH1 + CC*S1(2)
      RHO1S = DEN2 + CC*S1(1) + RHOWEG
       ERROR1 = ERROR2
      ERROR2 = RHO - RHODI - RHO1S
      IF( DABS(ERROR2).LT.1.D-6 ) GO TO 999
       DEN1 = HOLD
  44  CONTINUE
      IF(DABS(ERROR2).GT.1.D-6) WRITE (6,66) ERROR2,RHO,DTSTIN,DEN2,CC
  66  FORMAT(1X,'CONVER DOES NOT CONVERGE ',5E12.6)
  999 CONTINUE
      RETURN
      END
--------------------------------------------------------------------------------
      SUBROUTINE RTHETA(R,THETA, RHO,TEE )
C
CC   THIS VERSION WILL FIT DATA FOR THETA GT 1.0 AND LT 1.000001
CC   SOLVES     RHO = EM*THETA*(R**BETA)
CC   AND        TEE = R*(1.D0 BESQ*THETA*THETA)
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /COEFS/ A(20),Q(20)
      EQUIVALENCE (BETA,A(6)),(EM,A(7)),(BESQ,A(9))
C
      IF(EM.LE.0.D0 .OR. BESQ.LE.1.D0) GO TO 600
      ABSRHO = DABS( RHO )
      IF(ABSRHO .LT. 1.D-12 ) GO TO 600
      BEE = DSQRT(BESQ)
      IF(DABS(TEE) .LT. 1.D-12) GO TO 495
      IF( TEE .LT. 0.D0 ) Z = 1.D0-(1.D0-BEE)*TEE/(1.D0-BESQ)
     1  *(EM/ABSRHO)**(1.D0/BETA)
      IF(TEE.GT.0.D0) Z = (1.D0+TEE*(EM/BEE/ABSRHO)**(1.D0/BETA))**-BETA
      ~~IF( Z.GT.1.00234D0*BEE ) GO TO 496~~
      C = -RHO*BEE/EM/DABS(TEE)**BETA
      Z = DSIGN(Z,RHO)
 100  DO 500 N = 1, 16
      Z2 = Z*Z
      Z3 = 1.D0 - Z2
      DZ = Z3*(Z+C*DABS(Z3)**BETA)/(Z3+2.D0*BETA*Z2)
      Z = Z - DZ
      IF(DABS(DZ/Z) .LT. 1.D-12) GO TO 498
 500  CONTINUE
 601  WRITE(6,450) R,THETA, RHO,TEE,BETA,EM,BESQ
      IF(DABS(THETA).GT.1.0001) THETA=THETA/DABS(THETA)
        RETURN
  498 THETA = Z/BEE
      R = TEE/(1.D0-Z*Z)
      R = DABS(R)
        RETURN
  495 THETA = DSIGN(1.D0,RHO)/BEE
      R = (RHO/(EM*THETA))**(1.D0/BETA)
        RETURN
  496 THETA = DSIGN(1.,RHO)
        R     = TEE / ( 1.0D0 - BESQ )
      R = DABS(R)
        ~~RETURN~~
  600 IF(DABS(TEE) .LT. 1.D-12) GO TO 601
      IF(TEE.LT.0.D0) GO TO 496
        THETA = 1.D-12
        R     = TEE
 450  FORMAT(2X,'RTHETA DOES NOT CONVERGE' , 7(1X, E12.6) )
      RETURN
        END
--------------------------------------------------------------------------------
      SUBROUTINE SS(R,TH,S,SD)
C
      IMPLICIT DOUBLE PRECISION(A-H,O-Z)
      DIMENSION S(2),SD(2)
      COMMON /COEFS/ A(20),Q(20)
      EQUIVALENCE (ALPHA,Q(10)),(BETA,A(6)),(BESQ,A(9)),(DELTA,A(11))
     1,(DELI,Q(14)),(ALHI,Q(15)),(BETI,Q(16)),(GAMI,Q(17)),(P00,Q(11))
```

```
     2,(P01,Q(18)),(S00,A(17)),(S20,A(18)),(S01,A(19)),(S21,A(20))
C
      TT = TH*TH
      S(1)= S00 + S20*TT
      SD(1) = 2.*S20*TH
      S(2) = S01 + S21*TT
      SD(2) = 2.*S21*TH
      S(1)=S(1)*A(10)*A(7)*R**(1.-ALPHA)
      S(2)=S(2)*A(10)*A(12)*R**(1.-ALHI)
      RETURN
      END
--------------------------------------------------------------------------------
      SUBROUTINE AUX(R1,TH1,D2PDT2,D2PDMT,D2PDM2,AA,XK,SD,CVCOEX)
C
C  THIS SUBROUTINE CALCULATES SOME OF THE SECOND DERIVATIVES OF
C  THE ANOMALOUS PART OF THE EQUATION OF STATE.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON /COEFS/ A(20),Q(20)
      DIMENSION XK(2),S(2), SD(2),W(2),Y(2),Z(2),COEX(2)
      EQUIVALENCE (CC,A(1)),(BETA,A(6)),(BESQ,A(9)),(DELTA,A(11))
     1,(ALPHA,Q(10)),(S00,A(17)),(S20,A(18)),(S01,A(19)),(S21,A(20))
C
      DELI = 0.D0
      S(1)=S00+S20*TH1*TH1
      S(2)=S01+S21*TH1*TH1
      SD(1) = 2.*TH1*S20
      SD(2) = 2.*TH1*S21
      WW = 0.D0
      YY = 0.D0
      ZZ = 0.D0
      GAMMA = BETA*(DELTA - 1.D0)
      TT1 = TH1*TH1
      TER = 2.D0*BETA*DELTA - 1.D0
      G = (1.+(BESQ*TER-3.)*TT1 - BESQ*(TER-2.)*TT1*TT1)
      CVCOEX = 0.D0
      DO 30 I = 1,2
      ALHI = ALPHA - DELI
      BETI = BETA + DELI
      GAMI = GAMMA - DELI
      W(I)=(1.-ALHI)*(1.-3.*TT1)*S(I)-BETA*DELTA*(1.-TT1)*TH1*SD(I)
      W(I) = (W(I) *(R1**(-ALHI)))/G
      W(I) = W(I)*XK(I)
      WW = WW + W(I)
      Y(I) = BETI*(1.D0-3.D0*TT1)*TH1 - BETA*DELTA*(1.D0-TT1)*TH1
      Y(I) = (Y(I) *(R1**(BETI - 1.D0)))*XK(I)/G
      YY = YY + Y(I)
      Z(I) = 1.D0 - BESQ*(1.D0 - (2.D0*BETI))*TT1
      Z(I) = (Z(I) *(R1**(-GAMI)))*XK(I)/G
      ZZ = ZZ + Z(I)
      A1 = (BETA*(DELTA-3.D0)-3.D0*DELI-BESQ*ALHI*GAMI)
     1 /(2.D0*BESQ*BESQ*(2.D0-ALHI)*(1.D0-ALHI)*ALHI)
      A2 = 1+((BETA*(DELTA-3.D0)-3.D0*DELI-BESQ*ALHI*TER)/
     1  (2.D0*BESQ*(1.D0-ALHI)*ALHI))
      A2 = - A2
      A4 = 1.D0+((TER-2.D0)/(2.D0*ALHI))
      F1 = A1 + A2 + A4
      COEX(I) = ((2.D0 - ALHI)*(1.D0 - ALHI)*(R1**(-ALHI))*F1*XK(I))
      CVCOEX = CVCOEX + COEX(I)
      DELI = 0.5D0
 30   CONTINUE
      D2PDT2 = AA*WW
      D2PDMT = YY + AA*CC*WW
      D2PDM2 = ZZ/AA + 2.D0*CC*YY + (CC**2)*AA*WW
      RETURN
      END
--------------------------------------------------------------------------------
      SAMPLE MAIN PROGRAM
C
C  ISAT=-1 CALCULATES PROPERTIES ON THE VAPOR SIDE OF THE SATURATION CURVE.
C  ISAT=1 CALCULATES PROPERTIES ON THE LIQUID SIDE OF THE SATURATION CURVE.
C  ISAT=0 CALCULATES PROPERTIES AT ANY POINT ON THE THERMODYNAMIC SURFACE
C         NOT ON THE SATURATION CURVE.
C  IOPT=1 TAKES AS INPUT VARIABLES TEMPERATURE AND DENSITY (WHEN ISAT=0),
C         OR TEMPERATURE ONLY (WHEN ISAT.NE.0).
C  IOPT=2 TAKES AS INPUT VARIABLES TEMPERATURE AND PRESSURE (WHEN ISAT=0),
C         OR PRESSURE ONLY (WHEN ISAT.NE.0).
C  UNITS ARE AS FOLLOWS: TEMPERATURE IN (K), DENSITY IN (KG/M3), PRESSURE
C  IN (MPA), ENERGY IN (KJ/KG), SPECIFIC HEAT IN (KJ/KG.K), VELOCITY OF
C  SOUND IN (M/S), COMPRESSIBILITY IN (1/MPA), ETC.
C
      IMPLICIT REAL*8 (A-H,O-Z)
      REAL ANAMES,QNAMES
      COMMON/COEFS/A(20),Q(20),ANAMES(20),QNAMES(20)
      COMMON/THERM/IPHASE,DPDD,DPDT,U,H,S,CP,CV,CS,COMP
      COMMON/SATUR/ISAT,HEAT,DLIQ,DVAP
      COMMON/CRITS/TC,RHOC,PC,PCON,UCON,SCON,DPCON
```

```
      DATA DMIN,DMAX/200.D0,420.D0/
C
      CALL CONST
      WRITE(6,122)
      WRITE(6,159) (I,A(I),ANAMES(I),I,Q(I),QNAMES(I),I=1,20)
      WRITE(6,123)
C
 100  READ(5,1,END=999) ISAT,IOPT,T,D,P
      CALL RANGE(IRANGE,IOPT,T,D,P)
      IF(IRANGE.EQ.0) GO TO 100
      IF(IOPT.EQ.1) GO TO 200
      IF(ISAT.EQ.0) CALL DFIND(T,P,D,RHOC,DMIN,DMAX)
      IF(ISAT.NE.0) CALL TFIND(P,T)
 200  CALL THERMO(0,T,D,P,R,THETA)
      IF(ISAT.EQ.-1) WRITE(6,124) HEAT
      IF(ISAT.EQ. 1) WRITE(6,125) HEAT
      WRITE(6,121) IPHASE,T,D,P,U,H,S,CV,CP,CS,COMP
      GO TO 100
 999  STOP
C
 1    FORMAT( )
 159  FORMAT(5X,2HA(,I2,2H)=,F12.6,5X,A6,5X,2HQ(,I2,2H)=,F12.6,
     1   5X,A6)
 124  FORMAT(5X,'VAPOR SIDE OF SATURATION CURVE: LATENT HEAT=',
     1        F7.2)
 125  FORMAT(5X,'LIQUID SIDE OF SATURATION CURVE: LATENT HEAT=',
     1        F7.2)
 121  FORMAT(5X,'IN',I2,'-PHASE REGION :',3X,'T=',F8.3,3X,'D=',
     1   F8.3,3X,'P=',F8.4/5X,'U=',F7.1,3X,'H=',F7.1,3X,'S=',
     2   F7.4,3X,'CV=',F7.3/5X,'CP=',E10.5,3X,'CS=',F7.2,3X,
     3   'COMP=',E10.5/)
 122  FORMAT(1X,'PARAMETERS OF THE SCALED EQUATIONS ARE :'/)
 123  FORMAT(/' ENTER ISAT, IOPT, TEMPERATURE(K), DENSITY(KG/M3), ',
     1         'PRESSURE(MPA)'/' (ENTRY FOR DEPENDENT VARIABLE(S) WILL ',
     2         'BE IGNORED)'/)
      END
```

```
@XQT,F
 PARAMETERS OF THE SCALED EQUATIONS ARE :

     A( 1)=   -.017762     C          Q( 1)=   -.006000     POINTA
     A( 2)=   5.238000     P3         Q( 2)=   -.003000     POINTB
     A( 3)=    .000000     DELROC     Q( 3)=    .000000     DELPC
     A( 4)= -25.491500     P2         Q( 4)= 647.067000     TC
     A( 5)=   6.844500     P1         Q( 5)= 322.778000     RHOC
     A( 6)=    .325000     BETA       Q( 6)=  22.046000     PC
     A( 7)=   1.440300     KO         Q( 7)=    .267000     DPCDTC
     A( 8)=    .000000     DELTC      Q( 8)=  -1.600000     SLOPDI
     A( 9)=   1.375700     B*B        Q( 9)=    .491776     P11
     A(10)=  23.666600     A          Q(10)=    .108500     ALPHA
     A(11)=   4.820000     DELTA      Q(11)=    .586535     P00
     A(12)=    .294200     K1         Q(12)=  -1.026243     P20
     A(13)= -11.232600     MUC        Q(13)=    .612903     P40
     A(14)= -22.654700     MU1        Q(14)=    .500000     DELTAI
     A(15)= -17.887600     MU2        Q(15)=   -.391500     ALPHAI
     A(16)=  -4.933200     MU3        Q(16)=    .825000     BETA I
     A(17)=   1.109430     S00        Q(17)=    .741500     GAMMAI
     A(18)=  -1.981396     S20        Q(18)=    .103246     P01
     A(19)=    .246913     S01        Q(19)=    .160322     P21
     A(20)=   -.843411     S21        Q(20)=   -.169860     P41

 ENTER ISAT, IOPT, TEMPERATURE(K), DENSITY(KG/M3), PRESSURE(MPA)
 (ENTRY FOR DEPENDENT VARIABLE(S) WILL BE IGNORED)

> 0  1  680.  350.  0.
     IN 1-PHASE REGION :   T= 680.000   D= 350.000   P= 31.9201
     U= 2103.7   H= 2194.9   S= 4.5297   CV=  3.313
     CP= .21679+02   CS= 434.83   COMP= .98868-01

> 0  1  645.  240.  0.
     IN 2-PHASE REGION :   T= 645.000   D= 240.000   P= 21.5038
     U= 2143.7   H= 2233.3   S= 4.6405   CV= 13.729
     CP= .00000      CS=    .00   COMP= .00000

> 0  2  650.  0.  22.
  THIS POINT IS OUTSIDE THE RANGE!

> 0  2  650.  0.  23.
     IN 1-PHASE REGION :   T= 650.000   D= 390.593   P= 23.0000
     U= 1941.4   H= 2000.3   S= 4.2731   CV=  4.083
     CP= .94553+02   CS= 336.03   COMP= .52503+00

> -1  1  646.  0.  0.
     VAPOR SIDE OF SATURATION CURVE: LATENT HEAT= 277.94
     IN 1-PHASE REGION :   T= 646.000   D= 243.117   P= 21.7639
     U= 2150.1   H= 2239.6   S= 4.6485   CV=  5.421
     CP= .39360+03   CS= 324.40   COMP= .28377+01

> 1  2  0.  0.  21.8
     LIQUID SIDE OF SATURATION CURVE: LATENT HEAT= 264.78
     IN 1-PHASE REGION :   T= 646.138   D= 400.891   P= 21.8000
     U= 1912.7   H= 1967.1   S= 4.2266   CV=  4.585
     CP= .22126+03   CS= 298.08   COMP= .13549+01
```

Appendix E. Units and Conversion Factors

Units used in the tables

Pressure	MPa
Temperature	K
Density	kg/m^3
Energy, enthalpy, latent heat	kJ/kg
Entropy, specific heats	kJ/kg.K
Sound velocity	m/s

Conversion factors

Pressure

MPa to	multiply table values by
Pa	10^6
bar	10
atmosphere	9.869233
lbf/in^2	145.037738

Density

kg/m^3 to	multiply table values by
mol/dm^3	0.0555083
lbm/ft^3	0.0624280

Energy, enthalpy

kJ/kg to	multiply table values by
J/mol	18.01534
BTU/lbm	0.4299226

Entropy, specific heats

kJ/kg.K to	multiply table values by
J/mol.K	18.01534
BTU/lbm.R	0.2388459

Sound velocity

m/s to	multiply table value by
ft/s	3.28083

Temperature

K to	transform table value by
°C	$(T,°C) = (T,K) - 273.15$
°F	$(T,°F) = 1.8[(T,K) - 273.15] + 32$
R	$(T,R) = 1.8(T,K)$