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# Interpolation Correlations for Fluid Properties of Humid Air in the Temperature Range 100 °C to 200 °C

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This paper provides simple analytical correlations for selected thermodynamic and fluid transport properties for the mixture dry air and water vapor. These correlations are derived from theory as well as from numerical fitting procedures and give expressions for density  $\rho$ , viscosity  $\mu$ , thermal conductivity  $k$ , specific heat  $c_p$ , and Prandtl number  $Pr$  at a working pressure of  $p = 1$  bar and for a temperature range from 100 °C to 200 °C. The main purpose is to present a comparatively simple set of equations, as the correlations do not reflect in every case the underlying physical background. Since experimental data are scarce for the properties under investigation, it was in some cases necessary to extrapolate the available correlations to temperatures or water vapor contents where no experimental data could be found. The derived equations are compared with the pure component values for dry air and water vapor and, as far as possible, also for air-water vapor mixtures. © 1997 American Institute of Physics and American Chemical Society. [S0047-2689(97)00103-7]

Key words: density; humid air; humidity; Prandtl number; specific heat; thermal conductivity; viscosity.

## Contents

1. Introduction.....	1112	8. Coefficients for the calculation of viscosity $\mu$ of humid air according to Eq. (13).....	1117
2. Standard Evaluation Methods for Fluid Properties of Gas Mixtures.....	1113	9. Thermal conductivity ratio of humid air with respect to dry air $k/k_{\text{air}}$ according to data from Grüss and Schmick <sup>14</sup> .....	1117
2.1 Simple Mixing Correlations.....	1114	10. Thermal conductivity $k$ of humid air according to data from Grüss and Schmick (from Vargaftik <sup>27</sup> ).....	1118
2.2 One-Fluid Approximation.....	1115	11. Thermal conductivity $k$ of humid air according to data from Grüss and Schmick (from Touloukian <i>et al.</i> <sup>30</sup> ).....	1119
3. Density.....	1116	12. Thermal conductivity $k$ of a nitrogen/water vapor mixture according to experimental values from Timrot and Vargaftik (from Touloukian <i>et al.</i> <sup>30</sup> ).....	1119
4. Viscosity.....	1117	13. Coefficients for the calculation of the thermal conductivities of dry air and water vapor according to Eq. (17).....	1121
5. Thermal Conductivity.....	1118	14. Coefficients for the calculation of the temperature dependence of the specific heat of dry air and water vapor according to Eq. (22).....	1121
6. Specific Heat.....	1120	15. Coefficients for the calculation of the Prandtl number according to Eq. (24).....	1122
7. Prandtl Number.....	1120		
8. Application of Proposed Correlations.....	1121		
9. Conclusion.....	1123		
10. Acknowledgments.....	1123		
11. References.....	1123		

## List of Tables

1. Reference values for dry air and water vapor at ambient pressure $p = 1$ bar, as used in this paper <sup>3,10</sup> .....	1113
2. Composition of dry air according to the U.S. Standard Atmosphere <sup>16</sup> (from Wong and Embleton <sup>15</sup> ).....	1113
3. Compressibility factor $Z_A$ for dry air <sup>3</sup> and $Z_V$ for pure water vapor (calculated) <sup>10</sup> .....	1114
4. Coefficients used for the calculation of the compressibility factor of water vapor in the temperature range 100 °C to 200 °C.....	1114
5. Viscosity of humid air according to experimental values given by Kestin and Whitelaw <sup>25</sup> .....	1115
6. Viscosity of humid air according to experimental values given by Hochrainer and Munczak <sup>26</sup> .....	1116
7. Viscosity of humid air according to experimental	

## List of Figures

1. Density $\rho$ of humid air in the temperature range 100 °C to 200 °C according to Eqs. (10) and (12).....	1114
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2. Viscosity  $\mu$  of humid air in the temperature range 100 °C to 200 °C according to experimental values from Hochrainer and Munczak<sup>26</sup> and Studnikow (from Vargaftik<sup>27</sup>). . . . . 1117
3. Thermal conductivity ratio of humid air with respect to dry air  $k/k_{\text{air}}$ . Comparison of experimental values, a theoretical curve according to Grüss and Schmick,<sup>14</sup> the value for pure water vapor from 1928 and an extrapolation of Eqs. (14) to (19) down to a gas temperature of 80 °C. . . . . 1118
4. Thermal conductivity  $k$  of humid air in the temperature range 100 °C to 200 °C after Mason and Saxena [Eqs. (14) to (19)]. . . . . 1121
5. Specific heat  $c_p$  of humid air in the temperature range 100 °C to 200 °C according to Eq. (22). . . . . 1121
6. Prandtl number  $Pr$  of humid air in the temperature range 100 °C to 200 °C as calculated from Eq. (23) with the given correlations for the fluid properties. . . . . 1122
7. Relative increase of convective heat transfer with humidity. Comparison between experimental and theoretical values<sup>1</sup>. . . . . 1122
8. Comparison of the theoretically calculated heat transfer ratio with different approximations of the thermal conductivity  $k$  of the fluid. . . . . 1122

## 1. Introduction

Knowledge of the properties of humid air, including density, viscosity, thermal conductivity and specific heat, is important in many areas of engineering. Applications include building design, heating and air conditioning, humidification and drying, moisture separation, food processing and storage, and numerous manufacturing processes where a controlled humidity is necessary. The results of the present study are thus relevant in many fields of physics and engineering, although the investigations were motivated in the first instance by concern about the error in velocity measurements in moist air with a hot-wire anemometer if the effect of water vapor on the properties of the flowing medium is not considered.

In hot-wire anemometry fluid velocity is deduced from its influence on the heat transfer from a heated wire to the fluid. This heat transfer is governed both by parameters like fluid and wire temperature and geometrical dimensions, and by correlations describing the influence of the fluid properties and the flow field. These correlations need as an input the thermodynamic and transport parameters of the fluid. Most applications of hot-wire anemometry use air, and normally the properties of dry air are inserted in the correlations neglecting the influence of any water vapor present. At elevated temperatures, however, the water vapor partial pressure can be high enough to cause significant deviations in the heat transfer from the hot-wire and therefore also in the deduced velocity (see Durst *et al.*<sup>1</sup>). To evaluate the influence of humidity on the heat transfer from a hot-wire, the fluid

properties of the mixture are needed. Even though a comprehensive data collection for the density ( $\rho$ ), viscosity ( $\mu$ ), thermal conductivity ( $k$ ), and specific heat ( $c_p$ ) is available for the pure components water vapor and dry air, hardly any data for humid air are accessible.

Different approaches have been made to deal with the lack of experimental data for humid air. Simple attempts like a linear mixing of the values for dry air and water vapor (Krischer and Kast<sup>2</sup>) and also rather complicated and more general correlations like the one-fluid approximation (VDI-Wärmeatlas<sup>3</sup>) have been employed. The latter establishes from the critical data of the components a fictitious uniform gas with so-called pseudo-critical data from which the fluid properties of the real mixture can be derived. These approaches provide sufficient accuracy for most applications. However, if in a specific experiment the monitored signal shows a significant dependence on fluid properties, the one-fluid approach cannot be used, since for some properties of humid air deviations of up to 15% from the experimental value can occur within the temperature range considered in this paper (see Sec. 4).

The correlations and graphs presented in this paper are derived from a literature survey and should be regarded on the one hand as a compilation of the currently available data and on the other hand as a set of readily implemented functions for the properties under consideration. As far as experimental data were available, fitting-functions were defined to reproduce these data using the amount of water vapor and the temperature as input. In cases where no or only few data were at our disposal, theoretical approaches were tested for their capability to reproduce the subsidiary values for dry air and water vapor and also, if available, the experimental data of mixtures. The presented equations are to be understood as a practical aid to evaluating fluid properties, since the analytical form does not necessarily reflect the theoretical background. A set of more complex equations based on theoretical grounds can be found in Mason and Monchick.<sup>4</sup> However, their equations show higher deviations in comparison with the experimental values than the correlations derived in this paper. Various other publications, for example Hyland and Wexler,<sup>5,6</sup> present relationships for determining the specific volume and the enthalpy, from which the density and the specific heat respectively can be deduced. The viscosity and the thermal conductivity, however, are seldom given.

Since the accuracy of the input plays a prominent role for the investigation, the references were carefully selected:

- Dry air: The VDI-Wärmeatlas<sup>3</sup> gives a compilation of data from Baehr and Schwier,<sup>7</sup> Kadoya *et al.*<sup>8</sup> and Stephan and Laesecke.<sup>9</sup>

- Water vapor: Values were taken from Schmidt and Grigull,<sup>10</sup> respectively from the underlying IAPS Skeleton Tables,<sup>11</sup> the IAPS Formulation for the Viscosity,<sup>12</sup> and the IAPS Formulation for the Thermal Conductivity.<sup>13</sup>

Although the recommendations of Hyland and Wexler<sup>5,6</sup> are incorporated into the ASHRAE psychrometric charts, their formulations are complicated to compute. For the pure

TABLE 1. Reference values for dry air and water vapor at ambient pressure  $p=1$  bar, as used in this paper (Refs. 3 and 10).

Temperature (°C)	Density (kg/m <sup>3</sup> )	Viscosity (10 <sup>-6</sup> kg/m s)	Therm. conductivity (10 <sup>-3</sup> W/m K)	Specific heat (kJ/kg K)	Prandtl number
Dry air					
100	0.9329	21.94	31.39	1.012	0.7070
120	0.8854	22.80	32.75	1.014	0.7060
140	0.8425	23.65	34.08	1.016	0.7054
150	0.8226	24.07	34.74	1.018	0.7051
160	0.8036	24.48	35.39	1.019	0.7050
180	0.7681	25.29	36.68	1.022	0.7049
200	0.7356	26.09	37.95	1.026	0.7051
Water vapor					
100	0.5896	12.42	25.00	2.026	1.003
120	0.5577			2.005	
140	0.5294			1.991	
150	0.5165	14.29	28.90	1.986	0.978
160	0.5040			1.983	
180	0.4812			1.979	
200	0.4604	16.26	33.30	1.979	0.959

substances (air and water vapor) the reference values from the chosen sources are identical with those of References 5 and 6 within a tolerance of 0.1%. Table 1 shows the reference values for dry air and water vapor used throughout this article.

Section 2 of this paper describes conventional methods to determine analytically the fluid parameters. Beginning with an introduction to humidity definitions, terminology and important correlations applied in this paper, simple mixing correlations for the properties of the gas mixture humid air are given. This is followed by an approach according to the one-fluid approximation. Section 3 gives a formulation for the density ( $\rho$ ), which is basically a simple linear interpolation of the pure component values. The underlying equation for an ideal gas is expanded with the compressibility factor ( $Z$ ), representing a function of temperature and the amount of water vapor. Section 4 presents experimental data for the viscosity ( $\mu$ ) and compares different approaches with these data to deduce an analytical formulation. The thermal conductivity ( $k$ ) is dealt with in Sec. 5. Since for this quantity hardly any data are available, the fundamental shape of the correlation was derived from an early experiment<sup>14</sup> and the values of  $k$  were fixed by adapting the correlation to the pure component values of dry air and water vapor. Section 6 describes an analytical formulation for the specific heat ( $c_p$ ). Since no experimental data were found, a one-fluid approximation was made and from this the deviation in comparison with a linear mixing approach was examined. From the quantities already derived, the Prandtl number (Pr) is evaluated in Sec. 7 and a simpler correlation introduced. Section 8 illustrates the application of the proposed correlations to hot-wire anemometry and finally Sec. 9 summarizes the conclusions of the work.

## 2. Standard Evaluation Methods for Fluid Properties of Gas Mixtures

Different measures are available to describe the composition of a multi-component system, but in most cases the molar fraction  $x$  of the components is chosen. For the multi-component system of humid air considered in this paper, it is sufficient to determine the molar fraction of water vapor as long as a standard composition of dry air can be regarded as a pseudo-pure component. The correlations in the following sections are derived on the assumption that this concept is valid. A typical composition of air is given for example by the U.S. Standard Atmosphere, see Table 2 (from Wong and Embleton<sup>15</sup>).

The molar fraction  $x$  of water vapor, used throughout this paper, is a dimensionless, linear measure of humidity and has

TABLE 2. Composition of dry air according to the U.S. Standard Atmosphere (Ref. 16) [from Wong and Embleton (Ref. 15)].

Constituent gas and formula	Content (% by vol.)
Nitrogen (N <sub>2</sub> )	78.084
Oxygen (O <sub>2</sub> )	20.9476
Argon (Ar)	0.934
Carbon dioxide (CO <sub>2</sub> )	0.0314
Neon (Ne)	0.001818
Helium (He)	0.000524
Methane (CH <sub>4</sub> )	0.0002
Krypton (Kr)	0.000114
Hydrogen (H <sub>2</sub> )	0.00005
Nitrous oxide (N <sub>2</sub> O)	0.000027
Carbon monoxide (CO)	0.000019
Xenon (Xe)	0.0000087

TABLE 3. Compressibility factor  $Z_A$  for dry air (see Ref. 3) and  $Z_V$  for pure water vapor (calculated) (see Ref. 10).

$t$ (°C)	$Z_A$ ( $x=0$ )	$Z_V$ ( $x=1$ )
100	1.0000	0.9848
120	1.0000	0.9882
140	1.0000	0.9906
160	1.0000	0.9925
180	1.0000	0.9937
200	1.0000	0.9947

a straight-forward interpretation; it is used, for example, in the field of trace humidity, where the water vapor concentration is usually given as a volume concentration in [ppm<sub>V</sub>]. Since different application-specific measures are in use in engineering and science, a short introduction to some of the most important humidity measures and their conversion to molar fraction is given.

To define a basic measure for humid air, the traceability to SI units must be guaranteed, by defining either the mixing ratio  $r$  or the molar fraction  $x$ . The mixing ratio  $r$  is defined as the ratio of the mass of water vapor  $m_V$  with respect to the mass of dry air  $m_A$ :

$$r = \frac{m_V}{m_A} \quad (1)$$

In contrast to  $r$ , the molar fraction  $x$  is defined as the ratio of the number of moles of water vapor  $n_V$  with respect to the total number of moles of the mixture of humid air  $n_{HA}$ :

$$x = \frac{n_V}{n_{HA}} = \frac{n_V}{n_A + n_V} \quad (2)$$

Because in most cases humid air can be dealt with as an ideal gas, Dalton's Law

$$p = \sum_i p_i \quad (3)$$

can be applied. This implies that the total pressure  $p$  of the mixture can be obtained by summing the partial pressures  $p_i$  of all components  $i$ . So Eq. (2) can be rewritten for an ideal gas as

$$x = \frac{p_V}{p} \quad (4)$$

where  $p_V$  represents the water vapor partial pressure which is calculated from the measured humidity values (usually dewpoint temperature  $T_{DP}$  or relative humidity  $\varphi$ ). The dewpoint temperature is directly related to the water vapor partial pressure via the water vapor saturation correlation (Clausius Clapeyron), so that a conversion can be performed by using one of the formulations given for the vapor pressure of the pure water system, for example see Buck,<sup>17</sup> Sonntag<sup>18</sup> or Wexler.<sup>19</sup>

TABLE 4. Coefficients used for the calculation of the compressibility factor of water vapor in the temperature range 100 °C to 200 °C.

Coefficient	Value	Unit
$a$	1.007840	
$b$	$-3.4299543 \cdot 10^{-3}$	$\frac{1}{K}$
$c$	$-3.4396097 \cdot 10^{-3}$	$\frac{1}{K}$

Relative humidity  $\varphi$  is defined as the ratio of water vapor partial pressure  $p_V$  with respect to the saturation water vapor partial pressure  $p_{VS}$  at the same gas temperature.

$$\varphi = \frac{p_V}{p_{VS}} \quad (5)$$

Therefore, an additional temperature measurement is necessary to convert relative humidity  $\varphi$  to the water vapor partial pressure  $p_V$  by using the water vapor partial pressure formulations mentioned above for the calculation of the saturation value.

Using the calculated water vapor partial pressure, the molar fraction  $x$  can be calculated from Eq. (4) at a known ambient pressure  $p$ .

## 2.1 Simple Mixing Correlations

For most applications that take the fluid parameters of humid air into account (e.g., calculations considering drying processes or heat transfer in heat exchangers), simple mixing correlations are sufficient for a calculation of the fluid properties that are needed as input. The thermodynamic and fluid transport properties in this context are calculated from values given in standard tables for the pure components dry air and water vapor. Some of these fluid properties for the mixture can be derived by a simple linear combination according to the molar fraction of the components. Other properties need a more complex evaluation, because of intermolecular forces between molecules of one type as well as forces between

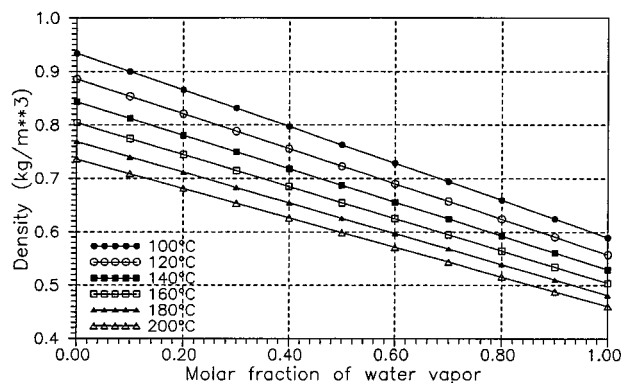


Fig. 1. Density  $\rho$  of humid air in the temperature range 100 °C to 200 °C according to Eqs. (10) and (12).

TABLE 5. Viscosity of humid air according to experimental values given by Kestin and Whitelaw (Ref. 25).

Temperature $t$ (°C)	$x_{\text{H}_2\text{O}}$	0.006	0.008	0.011	0.016	0.017	0.032	0.033		
25	$\mu$ ( $10^{-6} \frac{\text{kg}}{\text{m s}}$ )	18.451	18.446	18.441	18.419	18.200	18.399	18.374		
	$x_{\text{H}_2\text{O}}$	0.019	0.024	0.031	0.042	0.062	0.120			
50	$\mu$ ( $10^{-6} \frac{\text{kg}}{\text{m s}}$ )	19.593	19.591	19.575	19.539	19.474	19.247			
	$x_{\text{H}_2\text{O}}$	0.055	0.075	0.097	0.132	0.193	0.265	0.317	0.371	0.387
75	$\mu$ ( $10^{-6} \frac{\text{kg}}{\text{m s}}$ )	20.632	20.588	20.497	20.357	20.046	19.586	19.252	18.792	18.781
	$x_{\text{H}_2\text{O}}$									

molecules of two different types. Additionally, some of these equations differ when applications in different ranges of temperature and pressure are considered.

When humid air is treated as an ideal gas, its density  $\rho$  is often calculated using a linear mixing equation based upon the ideal gas law, according to the water vapor partial pressure  $p_v$ :

$$\rho = \frac{p}{RT} \frac{(p - p_v)M_A + p_v M_v}{p} \quad (6)$$

$p$  describes the ambient pressure,  $T$  the gas temperature,  $R$  the ideal gas constant,  $M_A$  and  $M_v$  the molecular masses of dry air and water vapor, respectively.

A similar linear mixing equation is used for thermal conductivity  $k$

$$k = k_A \left( 1 - \frac{p_v}{p} \right) + k_v \frac{p_v}{p}, \quad (7)$$

where  $k_A$  and  $k_v$  describe the thermal conductivities of dry air and water vapor, respectively.

Somewhat more complicated mixing correlations (see Krischer and Kast<sup>2</sup>) are used for the specific heat  $c_p$

$$c_p = \frac{c_{p,A}(p - p_v)M_A + c_{p,v}p_v M_v}{(p - p_v)M_A + p_v M_v} \quad (8)$$

and the viscosity  $\mu$

$$\mu = \frac{\mu_A(p - p_v)\sqrt{M_A} + \mu_v p_v \sqrt{M_v}}{(p - p_v)\sqrt{M_A} + p_v \sqrt{M_v}} \quad (9)$$

As before, the subscripts  $A$  and  $v$  indicate the values of the pure components dry air and water vapor.

For most applications the given equations are satisfactory in terms of accuracy for a calculation of the thermodynamic and transport properties of humid air. Nevertheless, a maximum error for the thermal conductivity of about 7% according to Krischer and Kast<sup>2</sup> and of about 10% as calculated by the authors occurs from a comparison with experimental data (see Sec. 5). For more sensitive applications, e.g., hot-wire anemometry where the effect of humidity on the heat transfer

is less than 6% at a gas temperature of 70 °C and a relative humidity range from 10% to 90%, this accuracy is not acceptable.

## 2.2. One-Fluid Approximation

An improvement with regard to a better understanding of the physical background of fluid mixtures is obtained by using evaluation methods based on the one-fluid (or random mixture) approximation. The one-fluid approximation does not combine the properties of the pure components of the mixture, but defines a fictitious single component fluid (so-called pseudo-pure) with appropriate pseudo-critical values (critical temperature, critical pressure, critical volume and dipole moment), which are calculated from the critical values of the mixture components. Using evaluation methods based on molecular theory the thermodynamic and transport properties of the mixture under investigation can be deduced.<sup>3,20-22</sup>

The derived equations according to this model can be found, e.g., in the VDI-Wärmeatlas<sup>3</sup> and show a good agreement for the pure component values for dry air and water vapor with the considered standard literature (see Sec. 1.); errors are in the range of a few percent. Nevertheless, in the mixture regime of humid air, errors up to 15% are found in comparison with experimental data (see the following sections for a more detailed description). Despite its comprehensiveness this model is not accurate enough for a calculation of fluid properties in applications where changing fluid composition induces measurement effects with a dynamic range of only a few percent. The deviations that occur for humid air (especially for the thermal conductivity and viscosity) indicate that a universal one-fluid model cannot represent the composition of humid air containing polar and nonpolar gases with sufficient accuracy. Nevertheless, especially in cases where no experimental data for a mixture are available at all, the one-fluid model usually leads to a good first estimate.

TABLE 6. Viscosity of humid air according to experimental values given by Hochrainer and Munczak (Ref. 26).

Temperature $t$ (°C)							
20	$x_{\text{H}_2\text{O}}$	0	0.0143	0.0190	0.0214		
	$\mu \left( 10^{-6} \frac{\text{kg}}{\text{m s}} \right)$	18.176	18.150	18.136	18.134		
30	$x_{\text{H}_2\text{O}}$	0	0.0171	0.0256	0.0341	0.0384	
	$\mu \left( 10^{-6} \frac{\text{kg}}{\text{m s}} \right)$	18.647	18.620	18.617	18.586	18.569	
40	$x_{\text{H}_2\text{O}}$	0	0.0148	0.0297	0.0456	0.0608	0.0676
	$\mu \left( 10^{-6} \frac{\text{kg}}{\text{m s}} \right)$	19.111	19.111	19.080	19.053	19.017	18.995
50	$x_{\text{H}_2\text{O}}$	0	0.0251	0.0499	0.0749	0.0998	0.1136
	$\mu \left( 10^{-6} \frac{\text{kg}}{\text{m s}} \right)$	19.588	19.553	19.483	19.426	19.363	19.343

### 3. Density

The density  $\rho$  of humid air is calculated in this work according to Eq. (6) which leads to the following correlation

$$\rho = \frac{1}{Z_{\text{HA}}(T,x)} \frac{p}{RT} \cdot [M_{\text{A}} \cdot (1-x) + M_{\text{V}} \cdot x], \quad (10)$$

where  $R$  is the ideal gas constant ( $R = 8.31451 \text{ J mol}^{-1} \text{ K}^{-1}$ ). In contrast to the ideal gas equation, the molecular mass is calculated from a linear combination of the molecular mass of dry air  $M_{\text{A}} = 28.9645 \text{ kg kmol}^{-1}$  and water vapor  $M_{\text{V}} = 18.01528 \text{ kg kmol}^{-1}$  according to the molar fraction  $x$  of the components. The compressibility factor  $Z_{\text{HA}}$  compensates for the real gas behavior and is dependent on temperature  $T$  and molar fraction  $x$ . Therefore, the accuracy of the equation depends directly on the accuracy of the compressibility factor.

There are different descriptions and approximations for the compressibility factor  $Z_{\text{HA}}$  available in literature. Several

methods based on molecular theory are given in Reid *et al.*<sup>22</sup> and a summary of equations deduced from the one-fluid approximation is given in Reference 3. An evaluation of the compressibility factor  $Z_{\text{HA}}$  according to the above mentioned equations results in an error of approximately 1% for the calculated density of the mixture dry air and pure water vapor in the temperature range 100 °C to 200 °C.

For the consideration of buoyancy effects on precision weighing experiments, the International Bureau of Weights and Measures (BIPM) employed a task group to formulate a correlation for the density of humid air (see Giacomo<sup>23</sup> and Davies<sup>24</sup>). Unfortunately, the validity of the given equation is restricted to a temperature range from 15 °C to 27 °C. Additionally, no values for pure water vapor can be evaluated since the relative humidity is used as an input to the correlations. The humidity in terms of water vapor partial pressure is, therefore, limited to the saturation water vapor pressure. Taking both aspects into account, this standard equation was discarded from further investigation.

TABLE 7. Viscosity of humid air according to experimental values given by Studnikow [from Vargaftik (Ref. 27)].

Temperature $t$ (°C)	$\mu \left( 10^{-6} \frac{\text{kg}}{\text{m s}} \right)$											
	$x_{\text{H}_2\text{O}}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
50		19.55	19.14									
60		20.01	19.60	19.04								
70		20.46	20.05	19.50	18.80							
80		20.91	20.51	19.95	19.25	18.43	17.50					
90		21.35	20.95	20.35	19.69	18.87	17.92	16.89	15.77			
100		21.80	21.40	20.84	20.14	19.31	18.36	17.32	16.18	14.99	13.75	12.47
110		22.23	21.84	21.28	20.58	19.75	18.79	17.74	16.60	15.39	14.13	12.84
120		22.66	22.27	21.72	21.01	20.18	19.22	18.16	17.01	15.79	14.52	13.21
130		23.09	22.71	22.16	21.45	20.61	19.64	18.58	17.42	16.19	14.90	13.58
140		23.50	23.12	22.58	21.87	21.03	20.06	18.98	17.84	16.58	15.29	13.95
150		23.92	23.55	23.01	22.31	21.47	20.49	19.42	18.24	16.99	15.68	14.33

TABLE 8. Coefficients for the calculation of viscosity  $\mu$  of humid air according to Eq. (13).

Coefficient	Value	Unit
$A_1$	6.0453459	$\frac{\text{kg}}{\text{m s}}$
$A_2$	0.042489943	$\frac{\text{kg}}{\text{m s K}}$
$B_1$	-6.8323022	$\frac{\text{kg}}{\text{m s}}$
$B_2$	0.0059284286	$\frac{\text{kg}}{\text{m s K}}$
$C_1$	-0.67799257	$\frac{\text{kg}}{\text{m s}}$
$C_2$	-0.011338714	$\frac{\text{kg}}{\text{m s K}}$

Since the internationally accepted standard formulations<sup>3,10</sup> provide values for the compressibility factor for dry air ( $Z_A$ ) and pure water vapor ( $Z_V$ ) in the range of consideration, these were taken as an input for a fitting procedure. Table 3 shows the values used in a restricted temperature range of 100 °C to 200 °C.

For dry air,  $Z_A$  remains constant; for water vapor a root mean square fit (rms) was performed leading to the equation

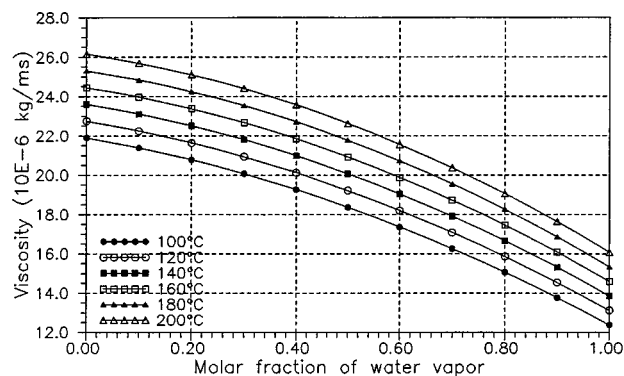
$$Z_V = \frac{a + cT}{1 + bT} \quad (11)$$

with coefficients given in Table 4.

At present, there is no agreed formulation to calculate the compressibility factor for the mixing regime. Since the values for dry air and water vapor differ by 1.5% at maximum in the temperature range required, a linear mixing model was applied for  $Z_{HA}$  according to the following equation

$$Z_{HA} = 1 + x \cdot \left( \frac{a + cT}{1 + bT} - 1 \right). \quad (12)$$

Equations (10) and (12) are used for a calculation of the density of humid air in the temperature range 100 °C to 200 °C; some calculated examples are shown in Fig. 1. Compared to the appropriate set, calculated according to Hyland and Wexler,<sup>5,6</sup> a maximum deviation of 0.36% occurs in the mixing regime; the subsidiary values are accurate to 0.1%.


 FIG. 2. Viscosity  $\mu$  of humid air in the temperature range 100 °C to 200 °C according to experimental values from Hochrainer and Munczak (Ref. 26) and Studnikow [from Vargaftik (Ref. 27)].

## 4. Viscosity

Some experimental values are available in the literature for the viscosity  $\mu$  of humid air in the temperature range from 20 °C to 150 °C. The values given by Kestin and Whitelaw,<sup>25</sup> Hochrainer and Munczak,<sup>26</sup> and Studnikow (from Vargaftik<sup>27</sup>) are shown in Tables 5 to 8.

Several models and approximations for the calculation of the viscosity  $\mu$  of humid air were tested by the authors. The calculated values were compared with the experimental values of Studnikow which were obtained at temperatures which are closest to the required temperature range. In general, the highest deviations were obtained in the middle of the humidity range ( $x \approx 0.5$ ):

- Linear mixing of pure component viscosities:

The resulting deviations, which lie in the range of 1% to 7% according to the experimental data from Studnikow, are better than expected considering the simplicity of this model.

- Calculation according to the one-fluid model<sup>3</sup>:

The deviations amount to 5% for the subsidiary values of pure water vapor. Within the mixing range errors up to 15% occur in comparison with the above cited experimental values. Also, the calculated values show a positive curvature with increasing molar fraction, whereas experimental values show negative curvature.

- Equation according to Nelson:<sup>28</sup>

 TABLE 9. Thermal conductivity ratio of humid air with respect to dry air  $k/k_{air}$  according to data from Grüss and Schmick (Ref. 14).

Temperature $t$ (°C)																		
80 (measured)	$x_{H_2O}$	0.000	0.071	0.150	0.171	0.197	0.225	0.250	0.306	0.312	0.444	0.519						
	$k/k_{air}$	1.000	1.020	1.035	1.037	1.036	1.035	1.037	1.026	1.030	0.999	0.974						
80 (fitted)	$x_{H_2O}$	0.000	0.071	0.150	0.171	0.197	0.225	0.250	0.306	0.312	0.444	0.519	0.600	0.700	0.800	0.900	1.000	
	$k/k_{air}$	1.000	1.021	1.032	1.033	1.033	1.033	1.032	1.028	1.027	1.006	0.990	0.970	0.942	0.913	0.882	0.850	



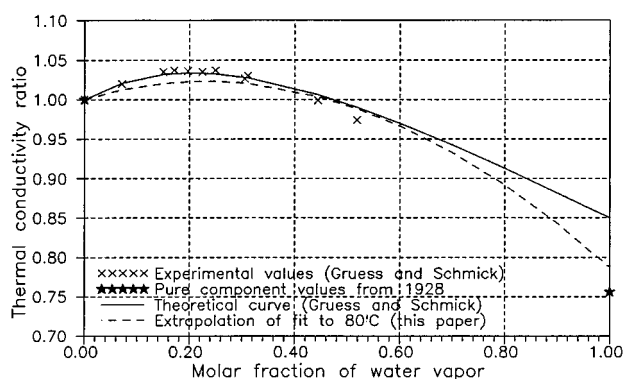


FIG. 3. Thermal conductivity ratio of humid air with respect to dry air  $k/k_{\text{air}}$ . Comparison of experimental values, a theoretical curve according to Grüss and Schmick (Ref. 14), the value for pure water vapor from 1928 and an extrapolation of Eqs. (14) to (19) down to a gas temperature of 80 °C.

This equation is specified only up to 100 °C gas temperature. A maximum deviation of about 4% occurs already at 100 °C in the middle of the mixing range, so that much higher deviations can be expected if the equation is used above the specified temperature range.

- Equation according to Wilke.<sup>29</sup>

This more complicated formulation results in a nearly linear behavior of the viscosity with increasing molar fraction (after Mason and Monchick<sup>4</sup>). An application of the equation in the form given by Kestin and Whitelaw<sup>25</sup> resulted in deviations of up to 14% from the experimental data in the humid regime.

- Equation given by Mason and Monchick:<sup>4</sup>

An evaluation according to this equation was carried out twice in the cited paper: In the first run theoretical input values according to kinetic gas theory and in the second run experimental values for the pure components were used. The second approach gives the better agreement of all presented approximations with the experimental data. The deviations over the complete mixing range of humid air do not exceed 2% to 3%.

An alternative method has been realized by the present authors: rather than using the formulation of Mason and Monchick which needs the viscosity as well as the diffusion constants of the pure components to be fitted by appropriate functions, the experimental data were fitted directly. A function was defined which is capable of reproducing the experimental data at all temperature levels with appropriate coefficients. In a second step these coefficients were fitted as

functions of temperature. In this way, a function for  $\mu$  with temperature and molar fraction of water vapor as input has been established for the temperature range 100 °C to 150 °C

$$\mu = [(A_1 + A_2 T) + (B_1 + B_2 T) \cdot x + (C_1 + C_2 T) \cdot x^2] \cdot 10^{-6}, \quad (13)$$

where the temperature  $T$  must be inserted in K. The resulting coefficients are given in Table 8.

Although input data are available only in the limited temperature range up to 150 °C, Eq. (13) is used over the whole range 100 °C to 200 °C. The calculated values are shown in Fig. 2. Above 150 °C there is hardly any change in slope and curvature of the viscosity, and the resulting values for the pure components agree within 1% deviation with the standard values given in References 3 and 10. This residual may be due to the deviation of approximately 1% of Studnikow's experimental data from today's standard values for the pure components.

## 5. Thermal Conductivity

Only a single measurement, published by Grüss and Schmick,<sup>14</sup> has been reported for the thermal conductivity  $k$  of humid air. They measured the thermal conductivity ratio of humid air with respect to the dry air value ( $k/k_{\text{air}}$ ) at a temperature of 80 °C (see Table 9 and Fig. 3). Additionally, a correlation based on the Wassiljewa equation was fitted to the data and is shown as the continuous curve in Fig. 3. The resulting values are also shown in Table 9.

Data for the thermal conductivity of pure water vapor that were available for Grüss and Schmick would have led to a ratio of  $k_{\text{water}}/k_{\text{air}} = 0.756$ . This is approximately the value that would be established from today's standard data, but they used a value of  $k_{\text{water}}/k_{\text{air}} = 0.850$  for the calculation of theoretical values to ensure the curvature and position of the maximum in thermal conductivity. The experimental data fit quite well over the range  $x = 0$  to  $x = 0.5$  (see Reference 14), but small positive deviations of the fitted values at the humid end with respect to the measurements indicate that the lower value for the conductivity ratio  $k_{\text{water}}/k_{\text{air}}$  is more appropriate. Moreover, an extrapolation of the equation given by Grüss and Schmick towards pure water vapor ( $x = 1$ ) shows large deviations from the reference values used in this paper. Different data can be found in the literature for the thermal conductivity of humid air, but in all cases the basic measurement of Grüss and Schmick is cited (see Tables 10 and 11).

TABLE 10. Thermal conductivity  $k$  of humid air according to data from Grüss and Schmick [from Vargaftik (Ref. 27)].

Temperature $t$ (°C)										
80	$x_{\text{H}_2\text{O}}$	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.0	
	$k \left( 10^{-3} \frac{\text{W}}{\text{m K}} \right)$	29.89	30.69	31.03	30.78	30.19	28.18	25.40	22.12	

TABLE 11. Thermal conductivity  $k$  of humid air according to data from Grüss and Schmick [from Touloukian *et al.* (Ref. 30)].

Temperature $t$ (°C)							
80	$x_{\text{H}_2\text{O}}$	0.000	0.197	0.306	0.444	0.519	1.000
	$k \left( 10^{-3} \frac{\text{W}}{\text{m K}} \right)$	28.69	29.92	29.61	28.85	28.14	21.90

The data from Tables 10 and 11 show a deviation of about 0.5% for dry air and about 4% for water vapor according to the reported standard values.<sup>3,10</sup> (This statement must be treated with caution: although  $k$  is hardly dependent on ambient pressure, the standard value for water vapor at 80 °C is taken from the saturation state at  $p = 473.73$  mbar, which is not the condition at which the measurement was carried out.) Although there are deviations in the absolute values, these data can be used to define qualitatively the slope and curvature of the thermal conductivity with increasing molar fraction. At low humidities  $k$  increases with molar fraction  $x$  and shows a relatively strong decrease after a maximum value at approximately  $x = 0.2$ .

Since dry air mainly consists of nitrogen, data for the thermal conductivity of the gas mixture nitrogen/water vapor can be regarded as a further source of information concerning the qualitative behavior of  $k$  for a varying amount of humidity (see Table 12). The maximum value becomes more prominent with increasing temperature and is also displaced towards higher values of molar fraction. Additionally, the lower value of  $k$  for pure water vapor increases with temperature and nearly approaches the value for dry air. The curvature and the behavior with changing temperature are typical for mixtures of a polar gas (water vapor) and a non-polar gas (dry air).<sup>22</sup> However, there are not enough experimental data to establish an approximating fit-function from experimental values for the whole temperature range from 100 °C to 200 °C. Therefore, a more theoretically based function had to be established for a fitting procedure.

In a first trial, the equations deduced from the one-fluid approximation were tested. The calculation of the thermal conductivity  $k$  resulted in deviations of up to 10% compared

with the subsidiary standard values. Additionally, neither the curvature of the experimental data nor the existence of a maximum value is represented by this correlation.

A good agreement with the experimental values for humid air from Grüss and Schmick (Table 9) was found using the equation of Mason and Saxena (from Reid *et al.*<sup>22</sup> or Touloukian *et al.*<sup>30</sup>). This equation is based on the more complicated Wassiljewa equation formulated by Lindsay and Bromley,<sup>31</sup> and shows the same structure as the equation given by Mason and Monchick<sup>4</sup> for calculation of the viscosity (see Sec. 4):

$$k = \frac{k_A}{1 + A_{A,V} \frac{x}{1-x}} + \frac{k_V}{1 + A_{V,A} \frac{1-x}{x}} \quad (14)$$

The required parameters  $A_{A,V}$  and  $A_{V,A}$  are calculated according to

$$A_{A,V} = \frac{\gamma}{2\sqrt{2}} \left( 1 + \frac{M_A}{M_V} \right)^{-0.5} \left[ 1 + \left( \frac{\mu_A M_V}{\mu_V M_A} \right)^{0.5} \left( \frac{M_A}{M_V} \right)^{0.25} \right]^2 \quad (15)$$

$$A_{V,A} = \frac{\gamma}{2\sqrt{2}} \left( 1 + \frac{M_V}{M_A} \right)^{-0.5} \left[ 1 + \left( \frac{\mu_V M_A}{\mu_A M_V} \right)^{0.5} \left( \frac{M_V}{M_A} \right)^{0.25} \right]^2 \quad (16)$$

The required pure component values for the thermal conductivities  $k_A$  for dry air and  $k_V$  for water vapor are calculated according to the standard values taken from References 3 and 10 using a third-order polynomial

$$k_i = A_i + B_i T + C_i T^2 + D_i T^3, \quad (17)$$

where the coefficients for dry air ( $i = A$ ) and water vapor ( $i = V$ ), respectively, are to be inserted as given in Table 13.

TABLE 12. Thermal conductivity  $k$  of a nitrogen/water vapor mixture according to experimental values from Timrot and Vargaftik [from Touloukian *et al.* (Ref. 30)].

Temperature $t$ (°C)									
65	$x_{\text{H}_2\text{O}}$	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.0
	$k \left( 10^{-3} \frac{\text{W}}{\text{m K}} \right)$	29.2	29.6	29.6	29.2	28.2	26.7	24.2	21.5
330	$x_{\text{H}_2\text{O}}$	0.0	0.1	0.2	0.3	0.4	0.6	0.8	1.0
	$k \left( 10^{-3} \frac{\text{W}}{\text{m K}} \right)$	46.7	50.1	52.0	52.6	52.4	51.0	49.1	47.0

As additional input parameters the viscosities for dry air  $\mu_A$  and water vapor  $\mu_V$  are evaluated according to Eq. (13):

$$\mu_A = (6.0453459 + 0.042489943 \cdot T) 10^{-6} \frac{\text{kg}}{\text{m s}}, \quad (18)$$

$$\mu_V = (-1.46494887 + 0.037079658 \cdot T) 10^{-6} \frac{\text{kg}}{\text{m s}}, \quad (19)$$

where the temperature  $T$  must be inserted in K.

According to Touloukian *et al.*,<sup>30</sup> for the empirical factor  $\gamma$  in Eqs. (15) and (16) a value of  $\gamma = 1.065$  should be used for nonpolar gases. For mixtures of polar and nonpolar gases a value of  $\gamma = 0.85$  is recommended. In this investigation, the lowest deviations between experimental data and theoretical predictions were obtained by using a slightly different value,  $\gamma = 0.80$ . The calculated values for the thermal conductivities  $k$  according to Eqs. (14) to (19) in a temperature range 100 °C to 200 °C are shown in Fig. 4.

The thermal conductivity calculated from the same equations at 80 °C is shown as the broken curve in Fig. 3, whose curvature satisfactorily reproduces the trend of the data. Although the maximum of the curve lies slightly below the data in the range up to  $x = 0.4$  the deviation for pure water vapor is notably lower than that predicted by the theoretical curve of Grüss and Schmick.

## 6. Specific Heat

According to kinetic gas theory it is sufficient to apply a simple linear mixing equation for the calculation of the specific heat  $c_p^{\text{id}}$  of a mixture of ideal gases. A mass-weighted equation is given, e.g., by Wong and Embleton<sup>15</sup>

$$c_p^{\text{id}} = \sum_i c_{p,i} x_i \frac{M_i}{M}, \quad (20)$$

where  $c_{p,i}$  denotes the value of the specific heat for the pure components  $c_{p,A}$  and  $c_{p,V}$ ,  $M_i$  the molar mass of the components  $i$  and  $M$  the mean molar mass of the gas mixture.

The one-fluid approximation proposes a correction term,  $\Delta c_p$ , which has to be added to the equation above to account for the real gas behavior of the mixture (for the calculation methods see, e.g., References 3 and 22)

$$c_p^{\text{re}} = c_p^{\text{id}} + \Delta c_p. \quad (21)$$

Besides the one-fluid approach, only the paper from Wong and Embleton<sup>15</sup> deals with the calculation of the specific heat of humid air in the temperature range 0 °C to 30 °C. An equation is given for the specific heat with the relative humidity as parameter, but applying this equation to higher temperatures results in increasing errors. The calculated values for dry air at a gas temperature of 200 °C show at least 10% deviation compared to the standard values of the specific heat. Additionally, the specific heat of pure water vapor cannot be calculated using the formulation for relative humidity.

For these reasons, instead of the correlation according to Wong and Embleton, Eq. (21) was evaluated for humid air. The correction term for the real gas behavior amounts to a maximum of 1.5% compared to a linear mixing approach. Since an uncertainty at this level is not significant relative to the uncertainties in  $\mu$  and  $k$ , this correction term was not considered further. In principle, any thermodynamic formulation for the specific enthalpy  $h$  could be used to determine  $c_p = dh/dT$ , but for the sake of simplicity and without compromising the accuracy of the final result a linear approach was employed.

A temperature dependent formulation of the specific heat for the pure components can be found for example in Touloukian and Makita,<sup>32</sup> but the results show deviations of some percent in the temperature range from 100 °C to 200 °C compared with the standard reference values.<sup>3,10</sup> Therefore, the specific heats for dry air and water vapor were evaluated from a fitting procedure of the given standard values using a third order polynomial. Subsequently, the specific heat  $c_p$  for humid air was derived by the following linear combination:

$$c_p = (A_A + B_A T + C_A T^2 + D_A T^3) \cdot (1 - x) + (A_V + B_V T + C_V T^2 + D_V T^3) \cdot x. \quad (22)$$

The respective coefficients are given in Table 14. The behavior of the specific heat  $c_p$  for humid air is shown in Fig. 5. The resulting deviations are in the range of 0.5% for water vapor and less than 0.05% for dry air when compared with the standard values for the pure components.

## 7. Prandtl Number

The dimensionless Prandtl number  $Pr$  is usually involved in correlations as a characteristic figure for fluid properties if any aspect of heat transfer is under investigation.

$$Pr = \frac{\nu}{\alpha} = \frac{c_p \mu}{k}, \quad (23)$$

where  $\alpha$  describes the thermal diffusivity  $\alpha = k/(\rho c_p)$ . The temperature and humidity dependence of  $Pr$  can be deduced by using the equations given in the preceding sections and is shown in Fig. 6.

To shorten the procedure involved in calculating  $c_p$ ,  $\mu$  and  $k$  individually, an equation was fitted to the calculated Prandtl number. The resulting correlation is valid in the temperature range from 100 °C to 200 °C

$$Pr = (A_1 + A_2 T + A_3 T^2) + (B_1 + B_2 T + B_3 T^2) \cdot x + (C_1 + C_2 T) \cdot x^2 + (D_1 + D_2 T)^{-1} \cdot x^3. \quad (24)$$

The coefficients are given in Table 15. This approximation leads to deviations of less than 1% in comparison with Prandtl numbers found in literature for dry air and water vapor.

TABLE 13. Coefficients for the calculation of the thermal conductivities of dry air and water vapor according to Eq. (17).

Coefficients for dry air		
$A_A$	$-0.56827429 \cdot 10^{-3}$	$\frac{W}{m K}$
$B_A$	$0.10805198 \cdot 10^{-3}$	$\frac{W}{m K^2}$
$C_A$	$-7.3956858 \cdot 10^{-8}$	$\frac{W}{m K^3}$
$D_A$	$3.7302922 \cdot 10^{-11}$	$\frac{W}{m K^4}$
Coefficients for water vapor		
$A_V$	$31.997566 \cdot 10^{-3}$	$\frac{W}{m K}$
$B_V$	$-0.13308958 \cdot 10^{-3}$	$\frac{W}{m K^2}$
$C_V$	$3.8160429 \cdot 10^{-7}$	$\frac{W}{m K^3}$
$D_V$	$-2.0 \cdot 10^{-10}$	$\frac{W}{m K^4}$

TABLE 14. Coefficients for the calculation of the temperature dependence of the specific heat of dry air and water vapor according to Eq. (22).

Coefficients for dry air		
$A_A$	1.0653697	$\frac{kJ}{kg K}$
$B_A$	$-4.4730851 \cdot 10^{-4}$	$\frac{kJ}{kg K^2}$
$C_A$	$9.8719042 \cdot 10^{-7}$	$\frac{kJ}{kg K^3}$
$D_A$	$-4.6376809 \cdot 10^{-10}$	$\frac{kJ}{kg K^4}$
Coefficients for water vapor		
$A_V$	6.564117	$\frac{kJ}{kg K}$
$B_V$	$-2.6905819 \cdot 10^{-2}$	$\frac{kJ}{kg K^2}$
$C_V$	$5.1820718 \cdot 10^{-5}$	$\frac{kJ}{kg K^3}$
$D_V$	$-3.2682964 \cdot 10^{-8}$	$\frac{kJ}{kg K^4}$

## 8. Application of Proposed Correlations

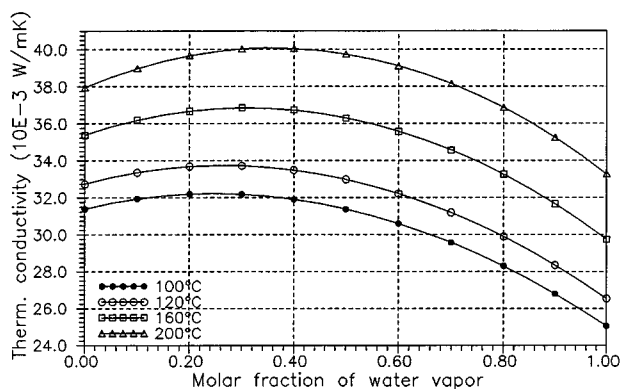
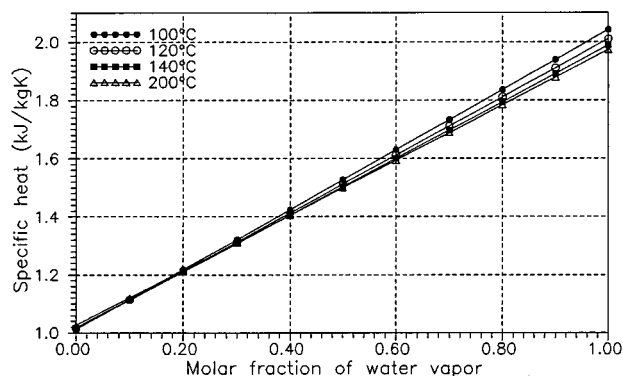
The validity of the calculated values has been tested indirectly by applying the derived correlations to estimate the influence of humidity on the heat transfer from a heated wire to the ambient fluid (see Durst *et al.*<sup>1</sup>).

The technique of hot-wire anemometry for gas velocity measurements uses the heat transfer coefficient,  $h$ , describing the heat transfer from the wire to the surrounding fluid, combined with the length,  $d$ , of the wire and the thermal conductivity,  $k$ , of the fluid to form the dimensionless Nusselt number,  $Nu = (hd)/k$ . This number represents all effects exerted

by fluid properties and the flow field on the heat transfer from the wire and, therefore, enables the flow velocity,  $U$ , to be deduced.

A wide variety of theoretically and semi-empirically derived equations is available for the Nusselt number, e.g., provided by an extensive theoretical treatment of heat transfer from cylinders. If only forced convection to the surrounding fluid is considered, all equations are of the type  $Nu = f(Re, Pr)$  or  $Nu = f(Pe, Pr)$ . The equations according to Kramers<sup>33</sup>

$$Nu = 0.42Pr^{0.20} + 0.57Pr^{0.33}Re^{0.50} \quad (25)$$


 FIG. 4. Thermal conductivity  $k$  of humid air in the temperature range 100 °C to 200 °C after Mason and Saxena [Eqs. (14) to (19)].

 FIG. 5. Specific heat  $c_p$  of humid air in the temperature range 100 °C to 200 °C according to Eq. (22).

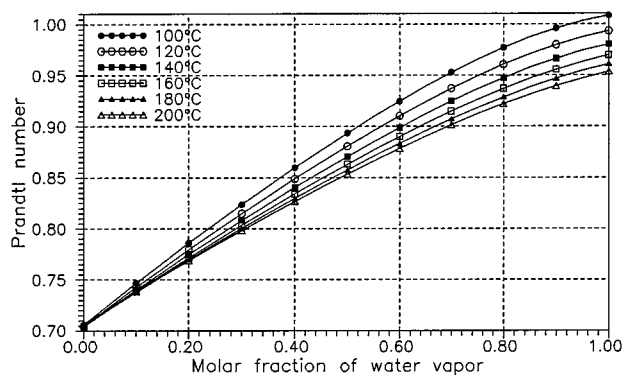


FIG. 6. Prandtl number  $Pr$  of humid air in the temperature range 100 °C to 200 °C as calculated from Eq. (23) with the given correlations for the fluid properties.

and Sucker and Brauer<sup>34</sup>

$$Nu = 0.462Pe^{0.1} + \frac{Pe^{0.7}}{1 + 2.79Pe^{0.2}} \cdot \frac{2.5}{[1 + (1.25Pr^{1/6})^{5/2}]^{2/5}} \quad (26)$$

are given as examples. The input parameters Reynolds number,  $Re = (Ud\rho)/\mu$ , Peclet number,  $Pe = (Udc_p\rho)/k$ , and Prandtl number,  $Pr = Pe/Re$ , are functions of the fluid velocity,  $U$ , and the fluid properties described in this paper. Nearly all equations for  $Nu$  use fluid properties evaluated at the film temperature, which is the arithmetic mean of fluid temperature and wire temperature.

The increase in heat transfer with increasing molar fraction  $x$  of water vapor at constant temperature and fluid velocity is traced to the variation of the fluid properties. Experimentally and theoretically derived values were compared

TABLE 15. Coefficients for the calculation of the Prandtl number according to Eq. (24).

Coefficient	Value	Unit
$A_1$	0.86681787	
$A_2$	$-7.7040097 \cdot 10^{-4}$	$\frac{1}{K}$
$A_3$	$9.1354464 \cdot 10^{-7}$	$\frac{1}{K^2}$
$B_1$	1.4841284	
$B_2$	$-4.5453245 \cdot 10^{-3}$	$\frac{1}{K}$
$B_3$	$4.38375 \cdot 10^{-6}$	$\frac{1}{K^2}$
$C_1$	0.057436032	
$C_2$	$-1.5429071 \cdot 10^{-4}$	$\frac{1}{K}$
$D_1$	23.198005	
$D_2$	$-0.090022367$	$\frac{1}{K}$

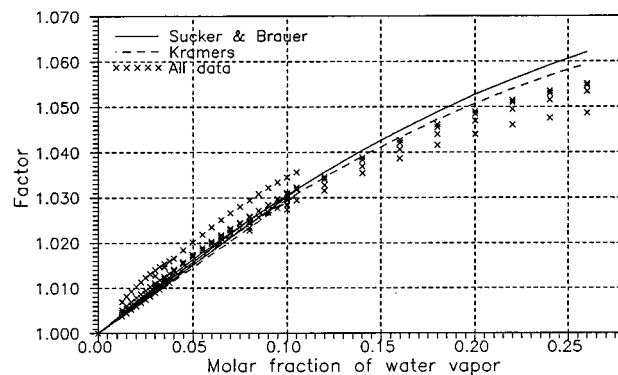


FIG. 7. Relative increase of convective heat transfer with humidity. Comparison between experimental and theoretical values (see Ref. 1).

by defining a comparative figure  $\varepsilon = \dot{Q}_x / \dot{Q}_{x=0}$  representing the ratio of heat transfer in humid air  $\dot{Q}_x$  to the respective value in dry air  $\dot{Q}_{x=0}$ . This procedure accounts for measurements at different temperatures and eliminates the effect of unquantified heat losses from the probe, see Durst *et al.*<sup>1</sup> Figure 7 shows results depicting  $\varepsilon$  as a function of the molar fraction  $x$  at three different temperatures (30 °C, 50 °C and 70 °C). For clarity of presentation, experimental results are shown with individual crosses rather than continuous curves. The symbols, however, represent curve fits to the data rather than the original data points. Theoretical curves according to the equations of Kramers and Sucker and Brauer are included for comparison. Both correlations were evaluated using humidity dependent formulations for the fluid parameters. Except for a small deviation at higher molar fractions  $x$  both approaches agree within the limits of the measuring accuracy with the experimental data.

For the application of hot-wire anemometry a correction factor for the gas velocity,  $U_x / U_{x=0}$ , could be more useful than  $\varepsilon$ . Such a correction (expressed as the velocity error resulting from neglecting the effect of humidity) has indeed been presented by Durst *et al.*<sup>1</sup> The evaluation of

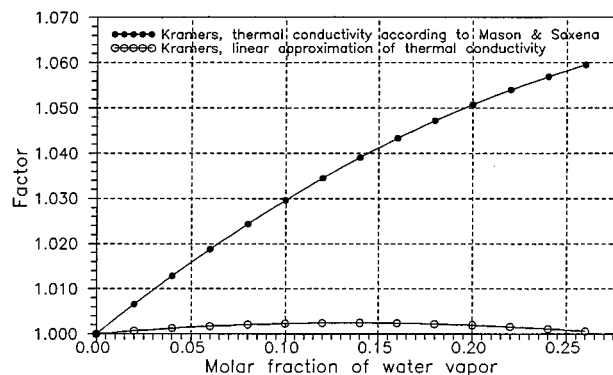


FIG. 8. Comparison of the theoretically calculated heat transfer ratio with different approximations of the thermal conductivity  $k$  of the fluid.

$U_x/U_{x=0}$ , however, depends on the equation for Nu chosen; the factor  $\varepsilon$  is, therefore, considered to be more fundamental and more relevant to the present paper.

To demonstrate the sensitivity of the Kramers and Sucker and Brauer correlations with regard to inaccurate fluid parameters, the correlation of Kramers was plotted with two different approaches for the thermal conductivity  $k$  leaving all other input values unchanged. Figure 8 shows the results for a formulation of  $k$  according to the presented correlation and according to a simple linear mixing correlation based on the subsidiary values of the pure components.

The example of the last two figures shows clearly that effects with a dynamic range of a few percent can be reproduced satisfactorily, using appropriate fluid parameters. The validity of the derived fluid parameters for humid air is thereby not proven, but their accuracy is demonstrated to be sufficient to predict small and complex effects.

## 9. Conclusion

This paper gives correlations for the density ( $\rho$ ), viscosity ( $\mu$ ), thermal conductivity ( $k$ ) and specific heat ( $c_p$ ) for a mixture of dry air and water vapor. The evaluated correlations are valid in a humidity range covering molar fractions from  $x=0$  to  $x=1$ , for a temperature range of  $100^\circ\text{C} < t < 200^\circ\text{C}$  and for an ambient pressure of  $p=1$  bar. The reference data were taken from internationally accepted standard formulations for the pure components as well as from the scarce experimental data for the mixing regime. From the deduced correlations for  $\rho$ ,  $\mu$ ,  $k$  and  $c_p$ , an additional polynomial is given for the Prandtl number covering the same range in terms of molar fraction of water vapor and temperature.

A thorough uncertainty analysis for the derived equations is hindered by the lack of precise experiments for the mixing regime of humid air. The absence of reference data was already recognized in 1965 by Mason and Monchick,<sup>4</sup> who recommended experiments especially for the determination of the viscosity and thermal conductivity of humid air, but this deficit has not been rectified in the intervening years. This paper shows that the requirement for data on  $\rho$ ,  $\mu$ ,  $k$  and  $c_p$  in water vapor/air mixtures remains urgent. A quantitative error analysis of the equations of the preceding sections could be realized for the pure component values for dry air ( $x=0$ ) and water vapor ( $x=1$ ). As far as possible, the deviations between the predictions and the experimental values have been quantified; in part the error approximations were taken from the cited authors.

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## 11. References

- <sup>1</sup>F. Durst, S. Noppenberger, M. Still, and H. Venzke, *Meas. Sci. Technol.* **7**, 1517 (1996).
- <sup>2</sup>O. Krischer and W. Kast, *Trocknungstechnik - Die wissenschaftlichen Grundlagen der Trocknungstechnik*, 3rd ed. (Springer, Berlin, Heidelberg, 1978), Vol. 1.
- <sup>3</sup>Verein Deutscher Ingenieure, *VDI Wärmeatlas: Berechnungsblätter für den Wärmeübergang*, 6th ed. (VDI, Düsseldorf, 1991).
- <sup>4</sup>E. A. Mason and L. Monchick, "Survey of the equation of state and transport properties of moist gases," in *Humidity and Moisture - Measurement and Control in Science and Industry*, Vol. 3, *Fundamentals and Standards*, edited by A. Wexler (Reinhold, New York, 1965).
- <sup>5</sup>R. W. Hyland and A. Wexler, *ASHRAE Trans.* **89**, (IIa) 500 (1983).
- <sup>6</sup>R. W. Hyland and A. Wexler, *ASHRAE Trans.* **89**, (IIa) 520 (1983).
- <sup>7</sup>H. D. Baehr and K. Schwier, *Die thermodynamischen Eigenschaften der Luft* (Springer, Berlin, 1961).
- <sup>8</sup>K. Kadoya, N. Matsunaga, and A. Nagashima, *J. Phys. Chem. Ref. Data* **14**, 947 (1985).
- <sup>9</sup>K. Stephan and A. Laesecke, *J. Phys. Chem. Ref. Data* **14**, 227 (1985).
- <sup>10</sup>E. Schmidt and U. Grigull, *Properties of Water and Steam in SI-Units: 0–800 °C, 0–1000 bar* (Springer, Berlin, 1989).
- <sup>11</sup>Release on the IAPS Skeleton Tables 1985 for the Thermodynamic Properties of Ordinary Water Substance, November 1985.
- <sup>12</sup>Release on the IAPS Formulation 1985 for the Viscosity of Ordinary Water Substance, November 1985.
- <sup>13</sup>Release on the IAPS Formulation 1985 for the Thermal Conductivity of Ordinary Water Substance, November 1985.
- <sup>14</sup>H. Grüss and H. Schmick, *Wiss. Veröffentlich. aus dem Siemens-Konzern* **7**, 202 (1928).
- <sup>15</sup>G. S. K. Wong and T. F. W. Embleton, *J. Acoust. Soc. Am.* **76**, 555 (1984).
- <sup>16</sup>U.S. Standard Atmosphere, U.S. G.P.O., Washington, DC (1976).
- <sup>17</sup>A. L. Buck, *J. Appl. Meteorology* **20**, 1527 (1981); revised 5/92 (personal communication).
- <sup>18</sup>D. Sonntag, *Z. Meteorol.* **70**, 340 (1990).
- <sup>19</sup>A. Wexler, *J. Res. Natl. Bur. Stand.* **80A**, 775 (1976).
- <sup>20</sup>T. W. Leland, J. S. Rowlinson, and G. A. Sather, *Trans. Faraday Soc.* **64**, 1447 (1968).
- <sup>21</sup>K. C. Mo and K. E. Gubbins, *Mol. Phys.* **31**, 825 (1976).
- <sup>22</sup>R. C. Reid, J. M. Prausnitz, and B. E. Poling, *The Properties of Gases and Liquids* (McGraw-Hill, New York, 1987).
- <sup>23</sup>P. Giacomo, *Metrologia* **18**, 33 (1982).
- <sup>24</sup>R. S. Davies, *Metrologia* **29**, 67 (1992).
- <sup>25</sup>J. Kestin and J. H. Whitelaw, "Measurements of the viscosity of dry and humid air," in *Humidity and Moisture - Measurement and Control in Science and Industry*, Vol. 3, *Fundamentals and Standards*, edited by A. Wexler (Reinhold, New York, 1965).
- <sup>26</sup>D. Hochrainer and F. Munczak, *Zähigkeitsmessungen von Luft verschiedener Feuchte und Temperatur*, Österreichische Akademie der Wissenschaften, Mathematisch-naturwissenschaftliche Klasse, Sitzungsberichte, Abteilung II **175**, 539 (1966).
- <sup>27</sup>N. B. Vargaftik, *Handbook of Physical Properties of Liquids and Gases - Pure Substances and Mixtures* (Hemisphere, Washington, 1983).
- <sup>28</sup>R. Nelson, *Chem. Eng. Prog.* **76**, 83 (1980).
- <sup>29</sup>C. R. Wilke, *J. Chem. Phys.* **18**, 517 (1950).
- <sup>30</sup>Y. S. Touloukian, P. E. Liley, and S. C. Saxena, *Thermal Conductivity. Nonmetallic Liquids and Gases* (IFI/Plenum Data Corporation, New York, 1970), Vol. 3.
- <sup>31</sup>A. L. Lindsay and L. A. Bromley, *Ind. Eng. Chem.* **42**, 1508 (1950).
- <sup>32</sup>Y. S. Touloukian and T. Makita, *Specific Heat. Nonmetallic Liquids and Gases* (IFI/Plenum Data Corporation, New York, 1970), Vol. 6.
- <sup>33</sup>H. Kramers, *Physica XII* **2-3**, 61 (1946).
- <sup>34</sup>D. Sucker and H. Brauer, *Wärme- und Stoffübertragung* **9**, 1 (1976).